

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise A, Question 1

Question:

Simplify each of the following expressions:

(a) $1 - \cos^2 \frac{1}{2}\theta$

(b) $5 \sin^2 3\theta + 5 \cos^2 3\theta$

(c) $\sin^2 A - 1$

(d) $\frac{\sin \theta}{\tan \theta}$

(e) $\frac{\sqrt{1 - \cos^2 x^\circ}}{\cos x^\circ}$

(f) $\frac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}}$

(g) $(1 + \sin x)^2 + (1 - \sin x)^2 + 2 \cos^2 x$

(h) $\sin^4 \theta + \sin^2 \theta \cos^2 \theta$

(i) $\sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

Solution:

(a) As $\sin^2 \frac{1}{2}\theta + \cos^2 \frac{1}{2}\theta \equiv 1$

So $1 - \cos^2 \frac{1}{2}\theta = \sin^2 \frac{1}{2}\theta$

(b) As $\sin^2 3\theta + \cos^2 3\theta \equiv 1$

So $5 \sin^2 3\theta + 5 \cos^2 3\theta = 5 (\sin^2 3\theta + \cos^2 3\theta) = 5$

(c) As $\sin^2 A + \cos^2 A \equiv 1$

So $\sin^2 A - 1 \equiv -\cos^2 A$

(d) $\frac{\sin \theta}{\tan \theta} = \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}}$

$= \sin \theta \times \frac{\cos \theta}{\sin \theta}$

$$= \cos \theta$$

$$(e) \frac{\sqrt{1 - \cos^2 x^\circ}}{\cos x^\circ} = \frac{\sqrt{\sin^2 x^\circ}}{\cos x^\circ} = \frac{\sin x^\circ}{\cos x^\circ} = \tan x^\circ$$

$$(f) \frac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}} = \frac{\sqrt{\sin^2 3A}}{\sqrt{\cos^2 3A}} = \frac{\sin 3A}{\cos 3A} = \tan 3A$$

$$\begin{aligned} (g) & (1 + \sin x^\circ)^2 + (1 - \sin x^\circ)^2 + 2 \cos^2 x^\circ \\ &= 1 + 2 \sin x^\circ + \sin^2 x^\circ + 1 - 2 \sin x^\circ + \sin^2 x^\circ + 2 \cos^2 x^\circ \\ &= 2 + 2 \sin^2 x^\circ + 2 \cos^2 x^\circ \\ &= 2 + 2 (\sin^2 x^\circ + \cos^2 x^\circ) \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

$$(h) \sin^4 \theta + \sin^2 \theta \cos^2 \theta = \sin^2 \theta (\sin^2 \theta + \cos^2 \theta) = \sin^2 \theta$$

$$(i) \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 = 1^2 = 1$$

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Exercise A, Question 2

Question:

Given that $2 \sin \theta = 3 \cos \theta$, find the value of $\tan \theta$.

Solution:

Given $2 \sin \theta = 3 \cos \theta$

So $\frac{\sin \theta}{\cos \theta} = \frac{3}{2}$ (divide both sides by $2 \cos \theta$)

So $\tan \theta = \frac{3}{2}$

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Exercise A, Question 3

Question:

Given that $\sin x \cos y = 3 \cos x \sin y$, express $\tan x$ in terms of $\tan y$.

Solution:

As $\sin x \cos y = 3 \cos x \sin y$

$$\text{So } \frac{\sin x \cos y}{\cos x \cos y} = 3 \frac{\cos x \sin y}{\cos x \cos y}$$

So $\tan x = 3 \tan y$

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Exercise A, Question 4

Question:

Express in terms of $\sin \theta$ only:

(a) $\cos^2 \theta$

(b) $\tan^2 \theta$

(c) $\cos \theta \tan \theta$

(d) $\frac{\cos \theta}{\tan \theta}$

(e) $(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)$

Solution:

(a) As $\sin^2 \theta + \cos^2 \theta \equiv 1$

So $\cos^2 \theta \equiv 1 - \sin^2 \theta$

(b) $\tan^2 \theta \equiv \frac{\sin^2 \theta}{\cos^2 \theta} \equiv \frac{\sin^2 \theta}{1 - \sin^2 \theta}$

(c) $\cos \theta \tan \theta$

$$= \cancel{\cos \theta} \times \frac{\sin \theta}{\cancel{\cos \theta}}$$

$$= \sin \theta$$

(d) $\frac{\cos \theta}{\tan \theta} = \frac{\cos \theta}{\frac{\sin \theta}{\cos \theta}} = \cos \theta \times \frac{\cos \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$

So $\frac{\cos \theta}{\tan \theta} = \frac{1 - \sin^2 \theta}{\sin \theta}$ or $\frac{1}{\sin \theta} - \sin \theta$

(e) $(\cos \theta - \sin \theta)(\cos \theta + \sin \theta) = \cos^2 \theta - \sin^2 \theta = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2\sin^2 \theta$

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Exercise A, Question 5

Question:

Using the identities $\sin^2 A + \cos^2 A \equiv 1$ and/or $\tan A \equiv \frac{\sin A}{\cos A}$ $\left(\cos A \neq 0 \right)$, prove that:

(a) $(\sin \theta + \cos \theta)^2 \equiv 1 + 2 \sin \theta \cos \theta$

(b) $\frac{1}{\cos \theta} - \cos \theta \equiv \sin \theta \tan \theta$

(c) $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

(d) $\cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$

(e) $(2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2 \equiv 5$

(f) $2 - (\sin \theta - \cos \theta)^2 \equiv (\sin \theta + \cos \theta)^2$

(g) $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \sin^2 x - \sin^2 y$

Solution:

(a) LHS = $(\sin \theta + \cos \theta)^2$
 $= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$
 $= (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta$
 $= 1 + 2 \sin \theta \cos \theta$
 = RHS

(b) LHS = $\frac{1}{\cos \theta} - \cos \theta$
 $= \frac{1 - \cos^2 \theta}{\cos \theta}$
 $= \frac{\sin^2 \theta}{\cos \theta}$
 $= \sin \theta \times \frac{\sin \theta}{\cos \theta}$
 $= \sin \theta \tan \theta$
 = RHS

(c) LHS = $\tan x^\circ + \frac{1}{\tan x^\circ}$
 $= \frac{\sin x^\circ}{\cos x^\circ} + \frac{\cos x^\circ}{\sin x^\circ}$
 $= \frac{\sin^2 x^\circ + \cos^2 x^\circ}{\sin x^\circ \cos x^\circ}$

$$= \frac{1}{\sin x^\circ \cos x^\circ}$$

$$= \text{RHS}$$

$$\begin{aligned} \text{(d) LHS} &= \cos^2 A - \sin^2 A \\ &\equiv \cos^2 A - (1 - \cos^2 A) \\ &\equiv \cos^2 A - 1 + \cos^2 A \\ &\equiv 2 \cos^2 A - 1 \checkmark \\ &\equiv 2(1 - \sin^2 A) - 1 \\ &\equiv 2 - 2 \sin^2 A - 1 \\ &\equiv 1 - 2 \sin^2 A \checkmark \end{aligned}$$

$$\begin{aligned} \text{(e) LHS} &= (2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2 \\ &\equiv 4 \sin^2 \theta - 4 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta + 4 \sin \theta \cos \theta + 4 \cos^2 \theta \\ &\equiv 5 \sin^2 \theta + 5 \cos^2 \theta \\ &\equiv 5(\sin^2 \theta + \cos^2 \theta) \\ &\equiv 5 \\ &\equiv \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(f) LHS} &\equiv 2 - (\sin \theta - \cos \theta)^2 \\ &= 2 - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta) \\ &= 2 - (1 - 2 \sin \theta \cos \theta) \\ &= 1 + 2 \sin \theta \cos \theta \\ &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= (\sin \theta + \cos \theta)^2 \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(g) LHS} &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y \\ &= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y \\ &= \sin^2 x - \sin^2 y \\ &= \text{RHS} \end{aligned}$$

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Exercise A, Question 6

Question:

Find, without using your calculator, the values of:

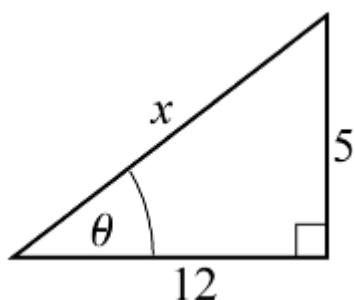
(a) $\sin \theta$ and $\cos \theta$, given that $\tan \theta = \frac{5}{12}$ and θ is acute.

(b) $\sin \theta$ and $\tan \theta$, given that $\cos \theta = -\frac{3}{5}$ and θ is obtuse.

(c) $\cos \theta$ and $\tan \theta$, given that $\sin \theta = -\frac{7}{25}$ and $270^\circ < \theta < 360^\circ$.

Solution:

(a)



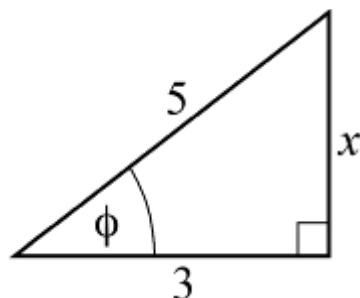
Using Pythagoras' Theorem,

$$x^2 = 12^2 + 5^2 = 169$$

$$x = 13$$

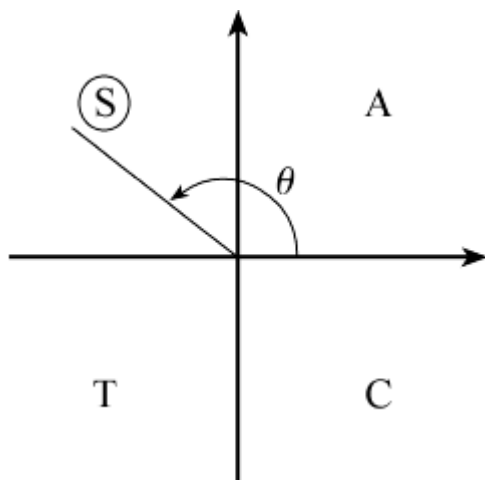
$$\text{So } \sin \theta = \frac{5}{13} \text{ and } \cos \theta = \frac{12}{13}$$

(b)



Using Pythagoras' Theorem, $x = 4$.

$$\text{So } \sin \phi = \frac{4}{5} \text{ and } \tan \phi = \frac{4}{3}$$



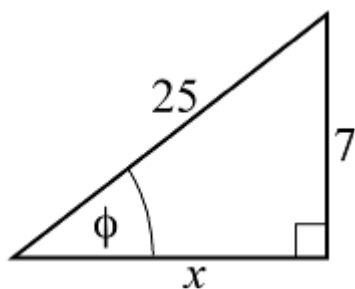
As θ is obtuse,

$$\sin \theta = \sin \phi = \frac{4}{5}$$

and

$$\tan \theta = -\tan \phi = -\frac{4}{3}$$

(c)



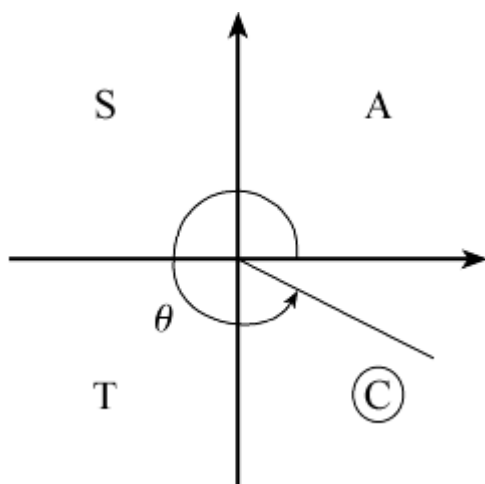
Using Pythagoras' Theorem,

$$x^2 + 7^2 = 25^2$$

$$x^2 = 25^2 - 7^2 = 576$$

$$x = 24$$

$$\text{So } \cos \phi = \frac{24}{25} \text{ and } \tan \phi = \frac{7}{24}$$



As θ is in the 4th quadrant,

$$\cos \theta = +\cos \phi = +\frac{24}{25}$$

and

$$\tan \theta = -\tan \phi = -\frac{7}{24}$$

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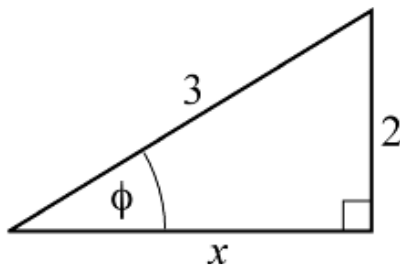
Exercise A, Question 7

Question:

Given that $\sin \theta = \frac{2}{3}$ and that θ is obtuse, find the exact value of: (a) $\cos \theta$, (b) $\tan \theta$.

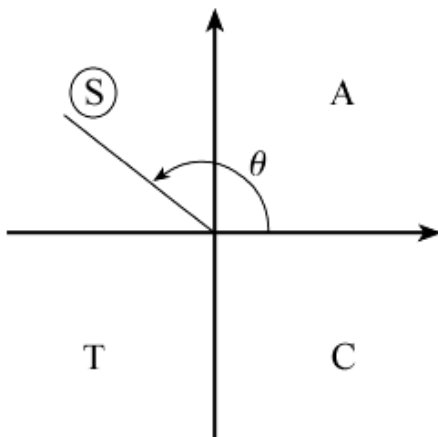
Solution:

Consider the angle ϕ where $\sin \phi = \frac{2}{3}$.



Using Pythagoras' Theorem, $x = \sqrt{5}$

(a) So $\cos \phi = \frac{\sqrt{5}}{3}$



As θ is obtuse, $\cos \theta = -\cos \phi = -\frac{\sqrt{5}}{3}$

(b) From the triangle,

$$\tan \phi = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Using the quadrant diagram,

$$\tan \theta = -\tan \phi = -\frac{2\sqrt{5}}{5}$$

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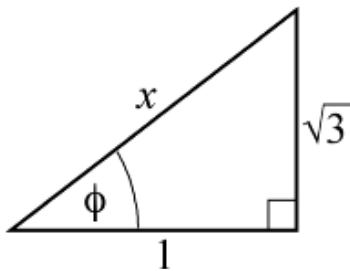
Exercise A, Question 8

Question:

Given that $\tan \theta = -\sqrt{3}$ and that θ is reflex, find the exact value of: (a) $\sin \theta$, (b) $\cos \theta$.

Solution:

Draw a right-angled triangle with $\tan \phi = +\sqrt{3} = \frac{\sqrt{3}}{1}$

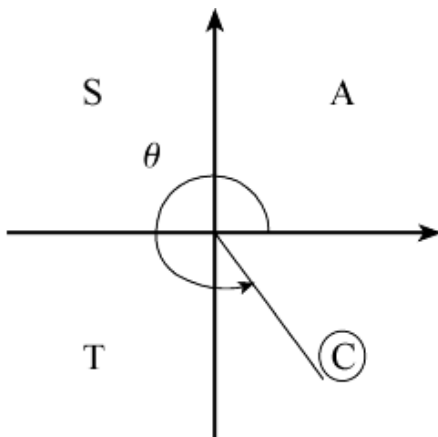


Using Pythagoras' Theorem,

$$x^2 = (\sqrt{3})^2 + 1^2 = 4$$

So $x = 2$

$$(a) \sin \phi = \frac{\sqrt{3}}{2}$$



As θ is reflex and $\tan \theta$ is $-ve$, θ is in the 4th quadrant.

$$\text{So } \sin \theta = -\sin \phi = \frac{-\sqrt{3}}{2}$$

$$(b) \cos \phi = \frac{1}{2}$$

$$\text{As } \cos \theta = \cos \phi, \cos \theta = \frac{1}{2}$$

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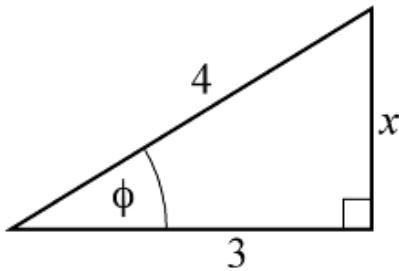
Exercise A, Question 9

Question:

Given that $\cos \theta = \frac{3}{4}$ and that θ is reflex, find the exact value of: (a) $\sin \theta$, (b) $\tan \theta$.

Solution:

Draw a right-angled triangle with $\cos \phi = \frac{3}{4}$



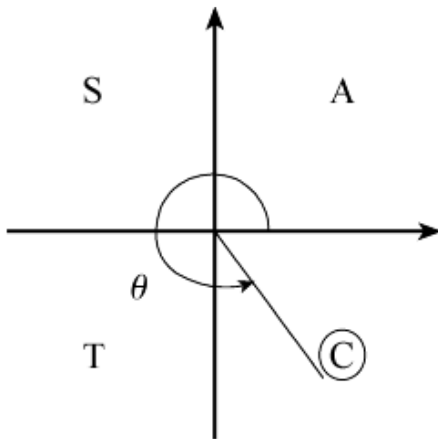
Using Pythagoras' Theorem,

$$x^2 + 3^2 = 4^2$$

$$x^2 = 4^2 - 3^2 = 7$$

$$x = \sqrt{7}$$

$$\text{So } \sin \phi = \frac{\sqrt{7}}{4} \text{ and } \tan \phi = \frac{\sqrt{7}}{3}$$



As θ is reflex and $\cos \theta$ is +ve, θ is in the 4th quadrant.

$$\text{(a) } \sin \theta = -\sin \phi = -\frac{\sqrt{7}}{4}$$

$$\text{(b) } \tan \theta = -\tan \phi = -\frac{\sqrt{7}}{3}$$

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Exercise A, Question 10

Question:

In each of the following, eliminate θ to give an equation relating x and y :

(a) $x = \sin \theta, y = \cos \theta$

(b) $x = \sin \theta, y = 2 \cos \theta$

(c) $x = \sin \theta, y = \cos^2 \theta$

(d) $x = \sin \theta, y = \tan \theta$

(e) $x = \sin \theta + \cos \theta, y = \cos \theta - \sin \theta$

Solution:

(a) As $\sin^2 \theta + \cos^2 \theta \equiv 1$
 $x^2 + y^2 = 1$

(b) $\sin \theta = x$ and $\cos \theta = \frac{y}{2}$

So, using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$x^2 + \left(\frac{y}{2}\right)^2 = 1 \text{ or } x^2 + \frac{y^2}{4} = 1 \text{ or } 4x^2 + y^2 = 4$$

(c) As $\sin \theta = x, \sin^2 \theta = x^2$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$x^2 + y = 1$$

(d) As $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$\cos \theta = \frac{\sin \theta}{\tan \theta}$$

So $\cos \theta = \frac{x}{y}$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$x^2 + \frac{x^2}{y^2} = 1 \text{ or } x^2 y^2 + x^2 = y^2$$

(e) $\sin \theta + \cos \theta = x$

$-\sin \theta + \cos \theta = y$

Adding up the two equations: $2 \cos \theta = x + y$

So $\cos \theta = \frac{x+y}{2}$

Subtracting the two equations: $2 \sin \theta = x - y$

So $\sin \theta = \frac{x-y}{2}$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$\left(\frac{x-y}{2}\right)^2 + \left(\frac{x+y}{2}\right)^2 = 1$$

$$x^2 - 2xy + y^2 + x^2 + 2xy + y^2 = 4$$

$$2x^2 + 2y^2 = 4$$

$$x^2 + y^2 = 2$$

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Exercise B, Question 1

Question:

Solve the following equations for θ , in the interval $0 < \theta \leq 360^\circ$:

(a) $\sin \theta = -1$

(b) $\tan \theta = \sqrt{3}$

(c) $\cos \theta = \frac{1}{2}$

(d) $\sin \theta = \sin 15^\circ$

(e) $\cos \theta = -\cos 40^\circ$

(f) $\tan \theta = -1$

(g) $\cos \theta = 0$

(h) $\sin \theta = -0.766$

(i) $7 \sin \theta = 5$

(j) $2 \cos \theta = -\sqrt{2}$

(k) $\sqrt{3} \sin \theta = \cos \theta$

(l) $\sin \theta + \cos \theta = 0$

(m) $3 \cos \theta = -2$

(n) $(\sin \theta - 1)(5 \cos \theta + 3) = 0$

(o) $\tan \theta = \tan \theta (2 + 3 \sin \theta)$

Solution:

(a) Using the graph of $y = \sin \theta$
 $\sin \theta = -1$ when $\theta = 270^\circ$

(b) $\tan \theta = \sqrt{3}$

The calculator solution is 60° ($\tan^{-1} \sqrt{3}$) and, as $\tan \theta$ is +ve, θ lies in the 1st and 3rd quadrants.
 $\theta = 60^\circ$ and $(180^\circ + 60^\circ) = 60^\circ, 240^\circ$

(c) $\cos \theta = \frac{1}{2}$

Calculator solution is 60° and as $\cos \theta$ is +ve, θ lies in the 1st and 4th quadrants.
 $\theta = 60^\circ$ and $(360^\circ - 60^\circ) = 60^\circ, 300^\circ$

(d) $\sin \theta = \sin 15^\circ$

The acute angle satisfying the equation is $\theta = 15^\circ$.
 As $\sin \theta$ is +ve, θ lies in the 1st and 2nd quadrants, so

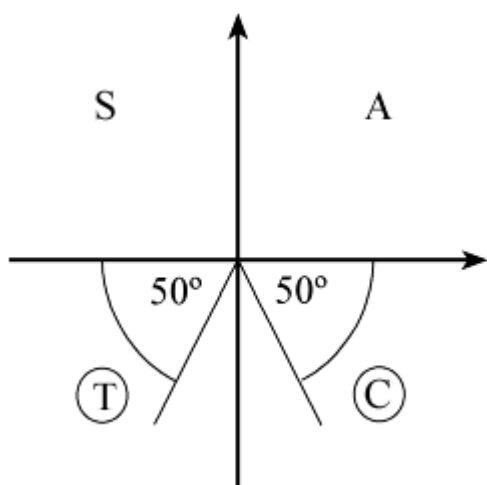
$$\theta = 15^\circ \text{ and } (180^\circ - 15^\circ) = 165^\circ$$

(e) A first solution is $\cos^{-1}(-\cos 40^\circ) = 140^\circ$
 A second solution of $\cos \theta = k$ is $360^\circ - 140^\circ$.
 So second solution is 220°
 (Use the quadrant diagram as a check.)

(f) A first solution is $\tan^{-1}(-1) = -45^\circ$
 Use the quadrant diagram, noting that as \tan is $-ve$, solutions are in the 2nd and 4th quadrants.
 (-45° is not in the given interval)
 So solutions are 135° and 315° .

(g) From the graph of $y = \cos \theta$
 $\cos \theta = 0$ when $\theta = 90^\circ, 270^\circ$

(h) The calculator solution is -50.0° (3 s.f.)
 As $\sin \theta$ is $-ve$, θ lies in the 3rd and 4th quadrants.



Solutions are 230° and 310° .

[These are $180^\circ + \alpha$ and $360^\circ - \alpha$ where $\alpha = \cos^{-1}(-0.766)$]

(i) $\sin \theta = \frac{5}{7}$

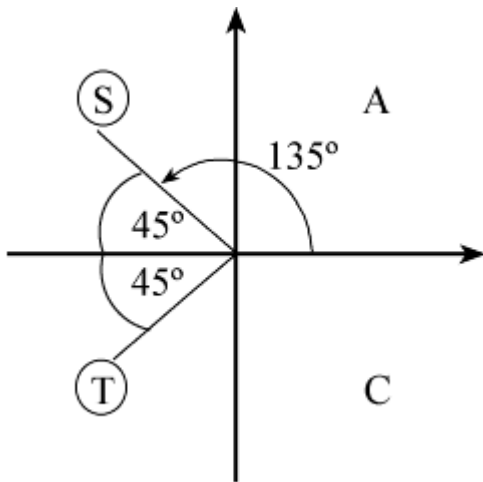
First solution is $\sin^{-1}\left(\frac{5}{7}\right) = 45.6^\circ$

Second solution is $180^\circ - 45.6^\circ = 134.4^\circ$

(j) $\cos \theta = -\frac{\sqrt{2}}{2}$

Calculator solution is 135°

As $\cos \theta$ is $-ve$, θ is in the 2nd and 3rd quadrants.



Solutions are 135° and 225° (135° and $360^\circ - 135^\circ$)

(k) $\sqrt{3} \sin \theta = \cos \theta$

So $\tan \theta = \frac{1}{\sqrt{3}}$ dividing both sides by $\sqrt{3} \cos \theta$

Calculator solution is 30°

As $\tan \theta$ is +ve, θ is in the 1st and 3rd quadrants.

Solutions are 30° , 210° (30° and $180^\circ + 30^\circ$)

(l) $\sin \theta + \cos \theta = 0$

So $\sin \theta = -\cos \theta \Rightarrow \tan \theta = -1$

Calculator solution (-45°) is not in given interval

As $\tan \theta$ is -ve, θ is in the 2nd and 4th quadrants.

Solutions are 135° and 315° [$180^\circ + \tan^{-1}(-1)$, $360^\circ + \tan^{-1}(-1)$]

(m) Calculator solution is $\cos^{-1}\left(-\frac{2}{3}\right) = 131.8^\circ$ (1 d.p.)

Second solution is $360^\circ - 131.8^\circ = 228.2^\circ$

(n) As $(\sin \theta - 1)(5 \cos \theta + 3) = 0$

either $\sin \theta - 1 = 0$ or $5 \cos \theta + 3 = 0$

So $\sin \theta = 1$ or $\cos \theta = -\frac{3}{5}$

Use the graph of $y = \sin \theta$ to read off solutions of $\sin \theta = 1$

$\sin \theta = 1 \Rightarrow \theta = 90^\circ$

For $\cos \theta = -\frac{3}{5}$,

calculator solution is $\cos^{-1}\left(-\frac{3}{5}\right) = 126.9^\circ$

second solution is $360^\circ - 126.9^\circ = 233.1^\circ$

Solutions are 90° , 126.9° , 233.1°

(o) Rearrange as

$\tan \theta (2 + 3 \sin \theta) - \tan \theta = 0$

$\tan \theta [(2 + 3 \sin \theta) - 1] = 0$ factorising

$\tan \theta (3 \sin \theta + 1) = 0$

So $\tan \theta = 0$ or $\sin \theta = -\frac{1}{3}$

From graph of $y = \tan \theta$, $\tan \theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ (0° not in given interval)

For $\sin \theta = -\frac{1}{3}$, calculator solution (-19.5°) is not in interval.

Solutions are $180^\circ - \sin^{-1}\left(-\frac{1}{3}\right)$ and $360^\circ + \sin^{-1}\left(-\frac{1}{3}\right)$ or use quadrant diagram.

Complete set of solutions $180^\circ, 199.5^\circ, 340.5^\circ, 360^\circ$

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Exercise B, Question 2

Question:

Solve the following equations for x , giving your answers to 3 significant figures where appropriate, in the intervals indicated:

(a) $\sin x^\circ = -\frac{\sqrt{3}}{2}, -180 \leq x \leq 540$

(b) $2 \sin x^\circ = -0.3, -180 \leq x \leq 180$

(c) $\cos x^\circ = -0.809, -180 \leq x \leq 180$

(d) $\cos x^\circ = 0.84, -360 < x < 0$

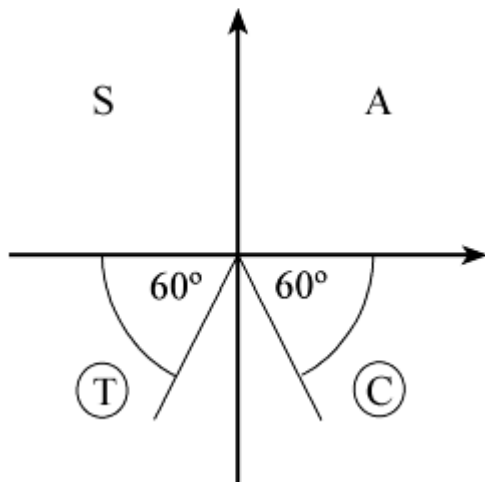
(e) $\tan x^\circ = -\frac{\sqrt{3}}{3}, 0 \leq x \leq 720$

(f) $\tan x^\circ = 2.90, 80 \leq x \leq 440$

Solution:

(a) Calculator solution of $\sin x^\circ = -\frac{\sqrt{3}}{2}$ is $x = -60$

As $\sin x^\circ$ is $-ve$, x is in the 3rd and 4th quadrants.



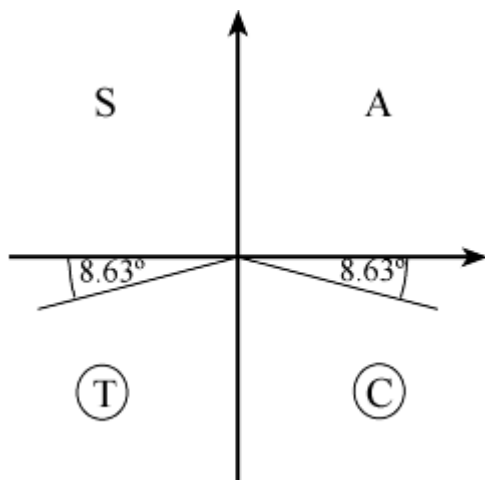
Read off all solutions in the interval $-180 \leq x \leq 540$
 $x = -120, -60, 240, 300$

(b) $2 \sin x^\circ = -0.3$

$\sin x^\circ = -0.15$

First solution is $x = \sin^{-1}(-0.15) = -8.63$ (3 s.f.)

As $\sin x^\circ$ is $-ve$, x is in the 3rd and 4th quadrants.

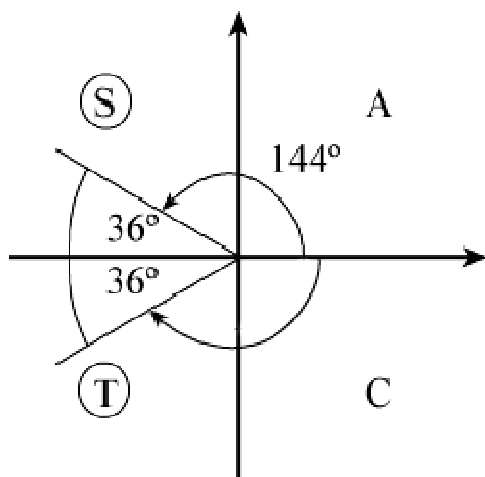


Read off all solutions in the interval $-180 \leq x \leq 180$
 $x = -171.37, -8.63 = -171, -8.63$ (3 s.f.)

(c) $\cos x^\circ = -0.809$

Calculator solution is 144 (3 s.f.)

As $\cos x^\circ$ is -ve, x is in the 2nd and 3rd quadrants.



Read off all solutions in the interval $-180 \leq x \leq 180$

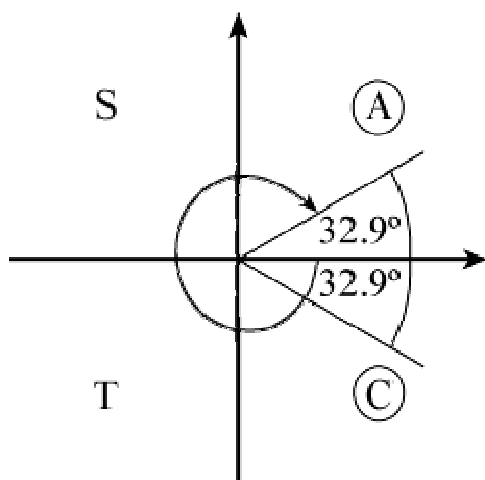
$x = -144, +144$

[Note: Here solutions are $\cos^{-1}(-0.809)$ and $\{360 - \cos^{-1}(-0.809)\} \{-360\}$

(d) $\cos x^\circ = 0.84$

Calculator solution is 32.9 (3 s.f.) (not in interval)

As $\cos x^\circ$ is +ve, x is in the 1st and 4th quadrants.



Read off all solutions in the interval $-360 < x < 0$

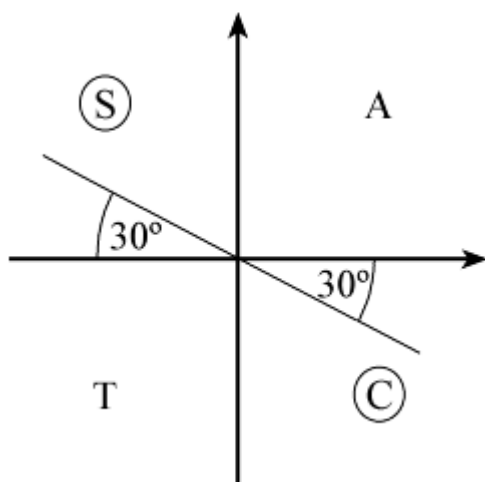
$$x = -327, -32.9 \text{ (3 s.f.)}$$

[Note: Here solutions are $\cos^{-1}(0.84) - 360$ and $\{360 - \cos^{-1}(0.84)\} - 360$]

$$(e) \tan x^\circ = -\frac{\sqrt{3}}{3}$$

$$\text{Calculator solution is } \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -30 \text{ (not in interval)}$$

As $\tan x^\circ$ is $-ve$, x is in the 2nd and 4th quadrants.



Read off all solutions in the interval $0 \leq x \leq 720$

$$x = 150, 330, 510, 690$$

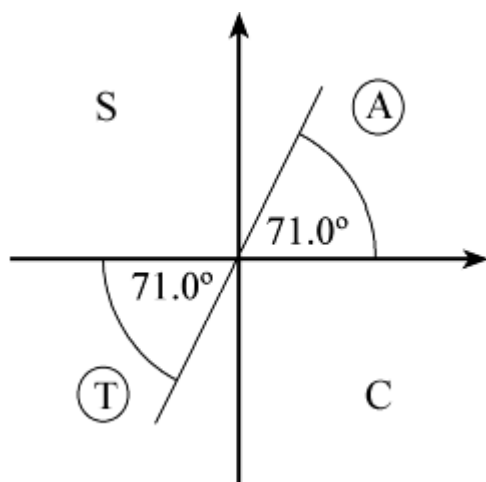
[Note: Here solutions are $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 180$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 360$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$$+ 540, \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 720]$$

$$(f) \tan x^\circ = 2.90$$

Calculator solution is $\tan^{-1}(2.90) = 71.0 \text{ (3 s.f.)}$ (not in interval)

As $\tan x^\circ$ is $+ve$, x is in the 1st and 3rd quadrants.



Read off all solutions in the interval $80 \leq x \leq 440$

$x = 251, 431$

[Note: Here solutions are $\tan^{-1}(2.90) + 180, \tan^{-1}(2.90) + 360$]

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise B, Question 3

Question:

Solve, in the intervals indicated, the following equations for θ , where θ is measured in radians. Give your answer in terms of π or 2 decimal places.

(a) $\sin \theta = 0, -2\pi < \theta \leq 2\pi$

(b) $\cos \theta = -\frac{1}{2}, -2\pi < \theta \leq \pi$

(c) $\sin \theta = \frac{1}{\sqrt{2}}, -2\pi < \theta \leq \pi$

(d) $\sin \theta = \tan \theta, 0 < \theta \leq 2\pi$

(e) $2(1 + \tan \theta) = 1 - 5 \tan \theta, -\pi < \theta \leq 2\pi$

(f) $2 \cos \theta = 3 \sin \theta, 0 < \theta \leq 2\pi$

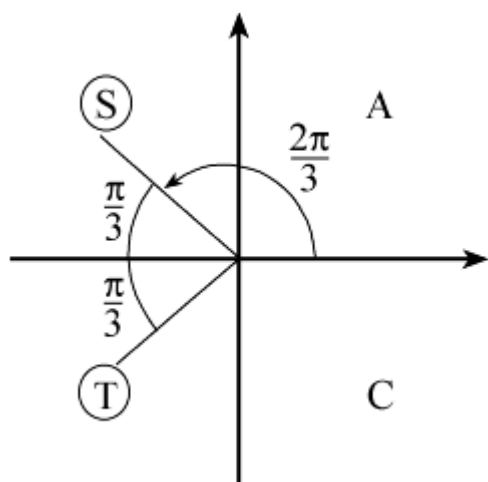
Solution:

(a) Use your graph of $y = \sin \theta$ to read off values of θ for which $\sin \theta = 0$.
In the interval $-2\pi < \theta \leq 2\pi$, solutions are $-\pi, 0, \pi, 2\pi$.

(b) Calculator solution of $\cos \theta = -\frac{1}{2}$ is $\cos^{-1} \left(-\frac{1}{2} \right) = 2.09$ radians

[You should know that $\cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3}$]

As $\cos \theta$ is $-ve$, θ is in 2nd and 3rd quadrants.

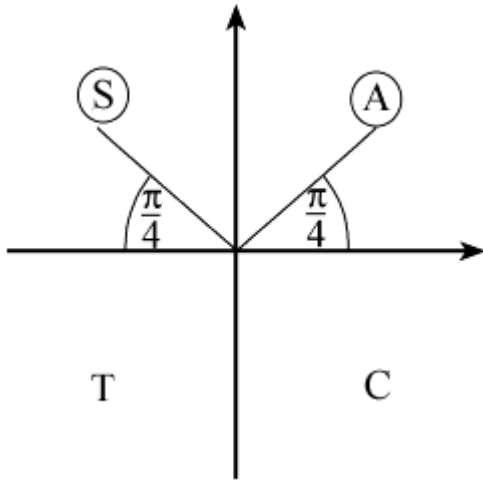


Read off all solutions in the interval $-2\pi < \theta \leq \pi$

$$\theta = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3} \quad (-4.19, -2.09, +2.09)$$

(c) Calculator solution of $\sin \theta = \frac{1}{\sqrt{2}}$ is $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = 0.79$ radians or $\frac{\pi}{4}$

As $\sin \theta$ is +ve, θ is in the 1st and 2nd quadrants.



Read off all solutions in the interval $-2\pi < \theta \leq \pi$

$$\theta = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

(d) $\sin \theta = \tan \theta$

$$\sin \theta = \frac{\sin \theta}{\cos \theta}$$

(multiply through by $\cos \theta$)

$$\sin \theta \cos \theta = \sin \theta$$

$$\sin \theta \cos \theta - \sin \theta = 0$$

$$\sin \theta (\cos \theta - 1) = 0$$

So $\sin \theta = 0$ or $\cos \theta = 1$ for $0 < \theta \leq 2\pi$

From the graph if $y = \sin \theta$, $\sin \theta = 0$ where $\theta = \pi, 2\pi$

From the graph of $y = \cos \theta$, $\cos \theta = 1$ where $\theta = 2\pi$

So solutions are $\pi, 2\pi$

(e) $2(1 + \tan \theta) = 1 - 5 \tan \theta$

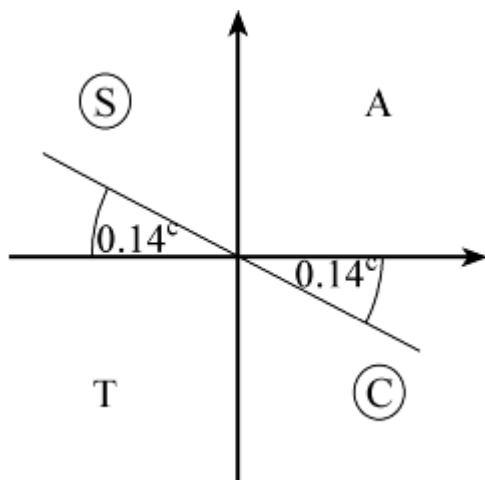
$$\Rightarrow 2 + 2 \tan \theta = 1 - 5 \tan \theta$$

$$\Rightarrow 7 \tan \theta = -1$$

$$\Rightarrow \tan \theta = -\frac{1}{7}$$

Calculator solution is $\theta = \tan^{-1} \left(-\frac{1}{7} \right) = -0.14$ radians (2 d.p.)

As $\tan \theta$ is -ve, θ is in the 2nd and 4th quadrants.



Read off all solutions in the interval $-\pi < \theta \leq 2\pi$

$$\theta = -0.14, 3.00, 6.14 \left[\tan^{-1} \left(-\frac{1}{7} \right), \tan^{-1} \left(-\frac{1}{7} \right) + \pi, \tan^{-1} \left(-\frac{1}{7} \right) + 2\pi \right]$$

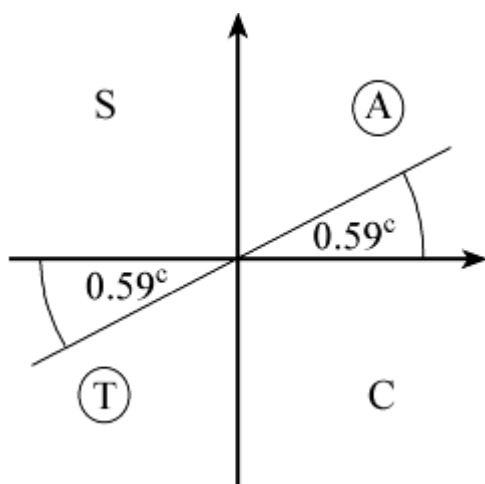
(f) As $2 \cos \theta = 3 \sin \theta$

$$\frac{2 \cos \theta}{3 \cos \theta} = \frac{3 \sin \theta}{3 \cos \theta}$$

$$\text{So } \tan \theta = \frac{2}{3}$$

Calculator solution is $\theta = \tan^{-1} \left(\frac{2}{3} \right) = 0.59$ radians (2 d.p.)

As $\tan \theta$ is +ve, θ is in the 1st and 3rd quadrants.



Read off all solutions in the interval $0 < \theta \leq 2\pi$

$$\theta = 0.59, 3.73 \left[\tan^{-1} \left(\frac{2}{3} \right), \tan^{-1} \left(\frac{2}{3} \right) + \pi \right]$$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise C, Question 1

Question:

Find the values of θ , in the interval $0 \leq \theta \leq 360^\circ$, for which:

(a) $\sin 4\theta = 0$

(b) $\cos 3\theta = -1$

(c) $\tan 2\theta = 1$

(d) $\cos 2\theta = \frac{1}{2}$

(e) $\tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}}$

(f) $\sin \left(-\theta \right) = \frac{1}{\sqrt{2}}$

(g) $\tan (45^\circ - \theta) = -1$

(h) $2 \sin (\theta - 20^\circ) = 1$

(i) $\tan (\theta + 75^\circ) = \sqrt{3}$

(j) $\cos (50^\circ + 2\theta) = -1$

Solution:

(a) $\sin 4\theta = 0 \quad 0 \leq \theta \leq 360^\circ$

Let $X = 4\theta$ so $0 \leq X \leq 1440^\circ$

Solve $\sin X = 0$ in the interval $0 \leq X \leq 1440^\circ$

From the graph of $y = \sin X$, $\sin X = 0$ where

$X = 0, 180^\circ, 360^\circ, 540^\circ, 720^\circ, 900^\circ, 1080^\circ, 1260^\circ, 1440^\circ$

$\theta = \frac{X}{4} = 0, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ, 360^\circ$

(b) $\cos 3\theta = -1 \quad 0 \leq \theta \leq 360^\circ$

Let $X = 3\theta$ so $0 \leq X \leq 1080^\circ$

Solve $\cos X = -1$ in the interval $0 \leq X \leq 1080^\circ$

From the graph of $y = \cos X$, $\cos X = -1$ where

$X = 180^\circ, 540^\circ, 900^\circ$

$\theta = \frac{X}{3} = 60^\circ, 180^\circ, 300^\circ$

(c) $\tan 2\theta = 1 \quad 0 \leq \theta \leq 360^\circ$

Let $X = 2\theta$

Solve $\tan X = 1$ in the interval $0 \leq X \leq 720^\circ$

A solution is $X = \tan^{-1} 1 = 45^\circ$

As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.

So $X = 45^\circ, 225^\circ, 405^\circ, 585^\circ$

$$\theta = \frac{X}{2} = 22 \frac{1}{2}^\circ, 112 \frac{1}{2}^\circ, 202 \frac{1}{2}^\circ, 292 \frac{1}{2}^\circ$$

$$(d) \cos 2\theta = \frac{1}{2} \quad 0 \leq \theta \leq 360^\circ$$

Let $X = 2\theta$

$$\text{Solve } \cos X = \frac{1}{2} \text{ in the interval } 0 \leq X \leq 720^\circ$$

$$\text{A solution is } X = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$$

As $\cos X$ is +ve, X is in the 1st and 4th quadrants.

So $X = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$$\theta = \frac{X}{2} = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

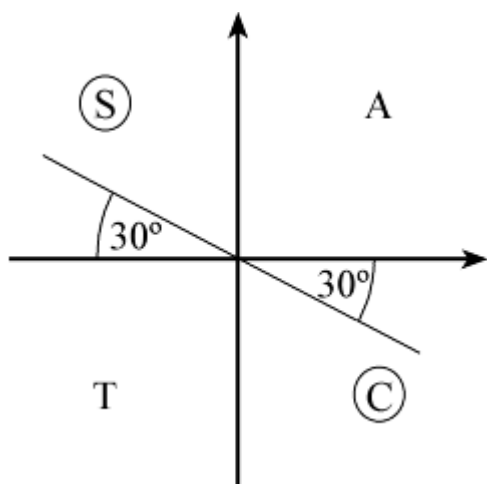
$$(e) \tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}} \quad 0 \leq \theta \leq 360^\circ$$

Let $X = \frac{1}{2}\theta$

$$\text{Solve } \tan X = -\frac{1}{\sqrt{3}} \text{ in the interval } 0 \leq X \leq 180^\circ$$

$$\text{A solution is } X = \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -30^\circ \text{ (not in interval)}$$

As $\tan X$ is -ve, X is in the 2nd and 4th quadrants.



Read off solutions in the interval $0 \leq X \leq 180^\circ$

$$X = 150^\circ$$

$$\text{So } \theta = 2X = 300^\circ$$

$$(f) \sin \left(-\theta \right) = \frac{1}{\sqrt{2}} \quad 0 \leq \theta \leq 360^\circ$$

Let $X = -\theta$

$$\text{Solve } \sin X = \frac{1}{\sqrt{2}} \text{ in the interval } 0 \geq X \geq -360^\circ$$

$$\text{A solution is } X = \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$$

As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.

$$X = -315^\circ, -225^\circ$$

So $\theta = -X = 225^\circ, 315^\circ$

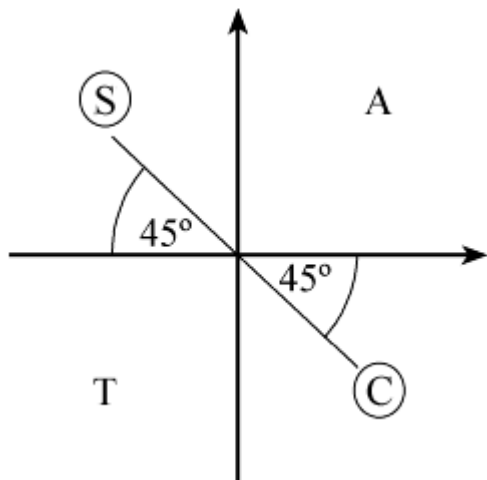
(g) $\tan(45^\circ - \theta) = -1$ $0 \leq \theta \leq 360^\circ$

Let $X = 45^\circ - \theta$ so $0 \geq -\theta \geq -360^\circ$

Solve $\tan X = -1$ in the interval $45^\circ \geq X \geq -315^\circ$

A solution is $X = \tan^{-1}(-1) = -45^\circ$

As $\tan X$ is -ve, X is in the 2nd and 4th quadrants.



$X = -225^\circ, -45^\circ$

So $\theta = 45^\circ - X = 90^\circ, 270^\circ$

(h) $2 \sin(\theta - 20^\circ) = 1$ so $\sin(\theta - 20^\circ) = \frac{1}{2}$ $0 \leq \theta \leq 360^\circ$

Let $X = \theta - 20^\circ$

Solve $\sin X = \frac{1}{2}$ in the interval $-20^\circ \leq X \leq 340^\circ$

A solution is $X = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

As $\sin X$ is +ve, solutions are in the 1st and 2nd quadrants.

$X = 30^\circ, 150^\circ$

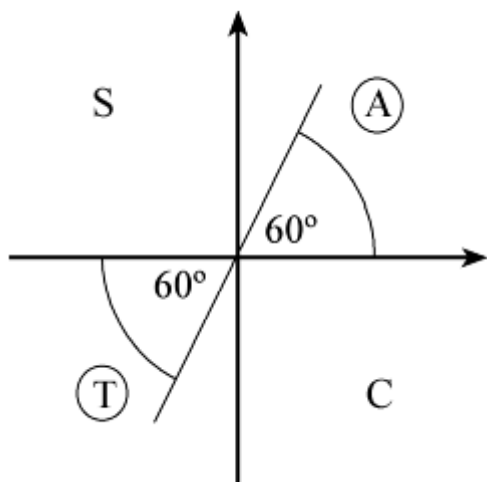
So $\theta = X + 20^\circ = 50^\circ, 170^\circ$

(i) Solve $\tan X = \sqrt{3}$ where $X = (\theta + 75^\circ)$

Interval for X is $75^\circ \leq X \leq 435^\circ$

One solution is $\tan^{-1}(\sqrt{3}) = 60^\circ$ (not in the interval)

As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



$$X = 240^\circ, 420^\circ$$

$$\text{So } \theta = X - 75^\circ = 165^\circ, 345^\circ$$

(j) Solve $\cos X = -1$ where $X = (50^\circ + 2\theta)$

Interval for X is $50^\circ \leq X \leq 770^\circ$

From the graph of $y = \cos X$, $\cos X = -1$ where

$$X = 180^\circ, 540^\circ$$

$$\text{So } 2\theta + 50^\circ = 180^\circ, 540^\circ$$

$$2\theta = 130^\circ, 490^\circ$$

$$\theta = 65^\circ, 245^\circ$$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise C, Question 2

Question:

Solve each of the following equations, in the interval given.
Give your answers to 3 significant figures where appropriate.

(a) $\sin \left(\theta - 10^\circ \right) = -\frac{\sqrt{3}}{2}, 0 < \theta \leq 360^\circ$

(b) $\cos (70 - x)^\circ = 0.6, -180 < x \leq 180$

(c) $\tan (3x + 25)^\circ = -0.51, -90 < x \leq 180$

(d) $5 \sin 4\theta + 1 = 0, -90^\circ \leq \theta \leq 90^\circ$

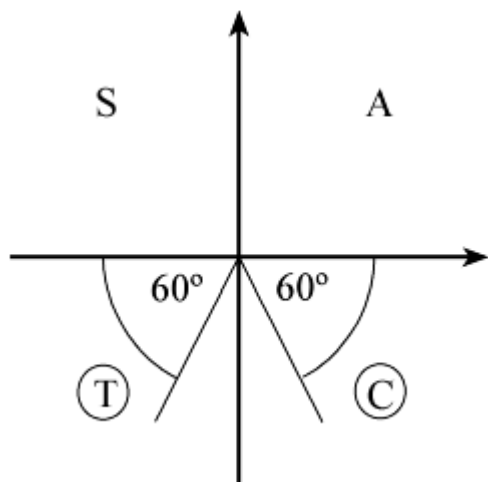
Solution:

(a) Solve $\sin X = -\frac{\sqrt{3}}{2}$ where $X = (\theta - 10^\circ)$

Interval for X is $-10^\circ < X \leq 350^\circ$

First solution is $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -60^\circ$ (not in interval)

As $\sin X$ is -ve, X is in the 3rd and 4th quadrants.



Read off solutions in the interval $-10^\circ < X \leq 350^\circ$

$X = 240^\circ, 300^\circ$

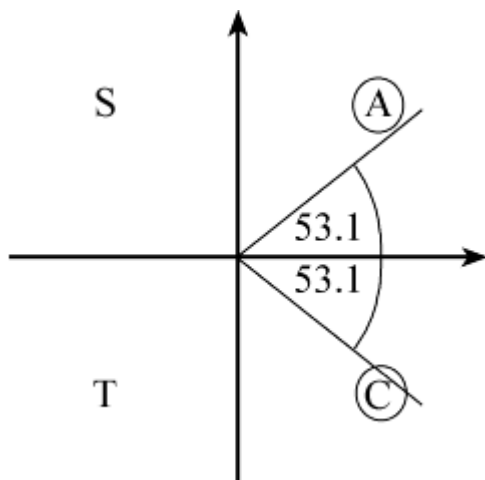
So $\theta = X + 10^\circ = 250^\circ, 310^\circ$

(b) Solve $\cos X^\circ = 0.6$ where $X = (70 - x)$

Interval for X is $180 + 70 > X \geq -180 + 70$ i.e. $-110 \leq X < 250$

First solution is $\cos^{-1} (0.6) = 53.1^\circ$

As $\cos X^\circ$ is +ve, X is in the 1st and 4th quadrants.



$$X = -53.1, +53.1$$

$$\text{So } x = 70 - X = 16.9, 123 \text{ (3 s.f.)}$$

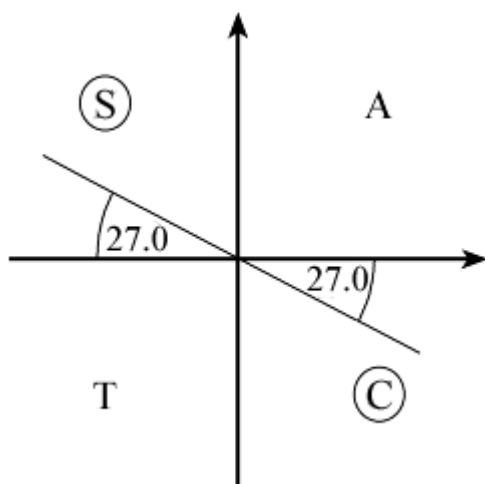
(c) Solve $\tan X^\circ = -0.51$ where $X = 3x + 25$

Interval for x is $-90 < x \leq 180$

So interval for X is $-245 < X \leq 565$

First solution is $\tan^{-1}(-0.51) = -27.0$

As $\tan X$ is -ve, X is in the 2nd and 4th quadrants.



Read off solutions in the interval $-245 < X \leq 565$

$$X = -207, -27, 153, 333, 513$$

$$3x + 25 = -207, -27, 153, 333, 513$$

$$3x = -232, -52, 128, 308, 488$$

$$\text{So } x = -77.3, -17.3, 42.7, 103, 163$$

(d) $5 \sin 4\theta + 1 = 0$

$$5 \sin 4\theta = -1$$

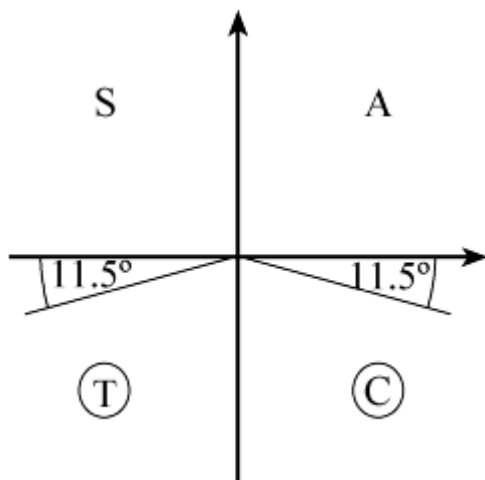
$$\sin 4\theta = -0.2$$

Solve $\sin X = -0.2$ where $X = 4\theta$

Interval for X is $-360^\circ \leq X \leq 360^\circ$

First solution is $\sin^{-1}(-0.2) = -11.5^\circ$

As $\sin X$ is -ve, X is in the 3rd and 4th quadrants.



Read off solutions in the interval $-360^\circ \leq X \leq 360^\circ$

$$X = -168.5^\circ, -11.5^\circ, 191.5^\circ, 348.5^\circ$$

$$\text{So } \theta = \frac{X}{4} = -42.1^\circ, -2.88^\circ, 47.9^\circ, 87.1^\circ$$

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Trigonometrical identities and simple equations

Exercise C, Question 3

Question:

Solve the following equations for θ , in the intervals indicated. Give your answers in radians.

$$(a) \sin \left(\theta - \frac{\pi}{6} \right) = -\frac{1}{\sqrt{2}}, \quad -\pi < \theta \leq \pi$$

$$(b) \cos (2\theta + 0.2^\circ) = -0.2, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$(c) \tan \left(2\theta + \frac{\pi}{4} \right) = 1, \quad 0 \leq \theta \leq 2\pi$$

$$(d) \sin \left(\theta + \frac{\pi}{3} \right) = \tan \frac{\pi}{6}, \quad 0 \leq \theta \leq 2\pi$$

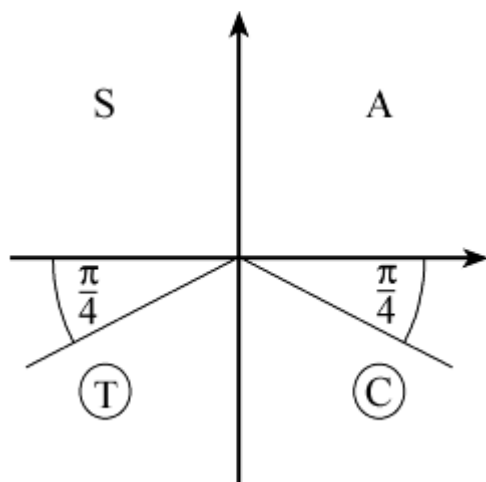
Solution:

$$(a) \text{ Solve } \sin X = -\frac{1}{\sqrt{2}} \text{ where } X = \theta - \frac{\pi}{6}$$

$$\text{Interval for } X \text{ is } -\frac{7\pi}{6} \leq X \leq \frac{5\pi}{6}$$

$$\text{First solution is } X = \sin^{-1} \left(-\frac{1}{\sqrt{2}} \right) = -\frac{\pi}{4}$$

As $\sin X$ is -ve, X is in the 3rd and 4th quadrants.



Read off solutions for X in the interval $-\frac{7\pi}{6} \leq X \leq \frac{5\pi}{6}$

$$X = -\frac{3\pi}{4}, \quad -\frac{\pi}{4}$$

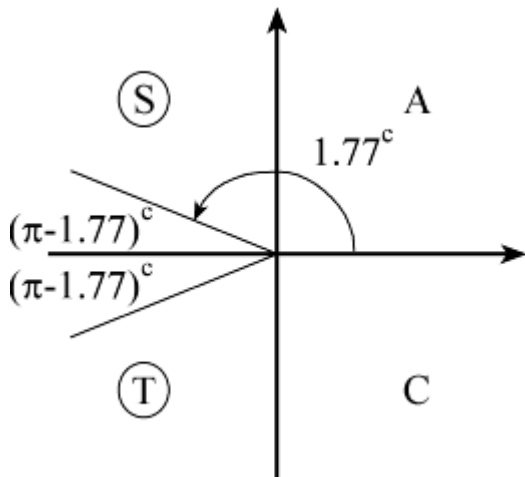
$$\text{So } \theta = X + \frac{\pi}{6} = \frac{\pi}{6} - \frac{3\pi}{4}, \frac{\pi}{6} - \frac{\pi}{4} = -\frac{7\pi}{12}, -\frac{\pi}{12}$$

(b) Solve $\cos X = -0.2$ where $X = 2\theta + 0.2$ radians

Interval for X is $-\pi + 0.2 \leq X \leq \pi + 0.2$ i.e. $-2.94 \leq X \leq 3.34$

First solution is $X = \cos^{-1}(-0.2) = 1.77 \dots$ radians

As $\cos X$ is $-ve$, X is in the 2nd and 3rd quadrants.



Read off solutions for X in the interval $-2.94 \leq X \leq 3.34$

$$X = -1.77, +1.77 \text{ radians}$$

$$2\theta + 0.2 = -1.77, +1.77$$

$$2\theta = -1.97, +1.57$$

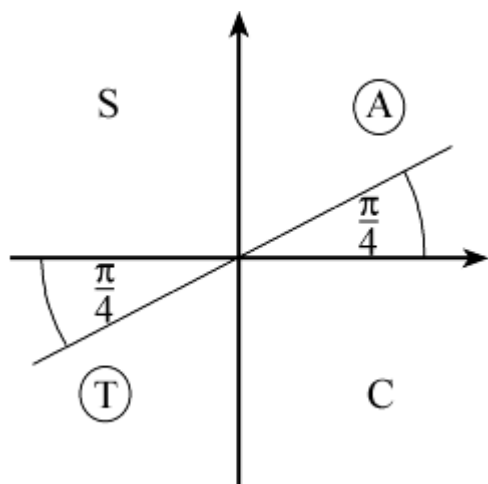
$$\text{So } \theta = -0.986, 0.786$$

(c) Solve $\tan X = 1$ where $X = 2\theta + \frac{\pi}{4}$

Interval for X is $\frac{\pi}{4} \leq X \leq \frac{17\pi}{4}$

First solution is $X = \tan^{-1} 1 = \frac{\pi}{4}$

As \tan is $+ve$, X is in the 1st and 3rd quadrants.



Read off solutions in the interval $\frac{\pi}{4} \leq X \leq \frac{17\pi}{4}$

$$X = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$$

$$2\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$$

$$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$$

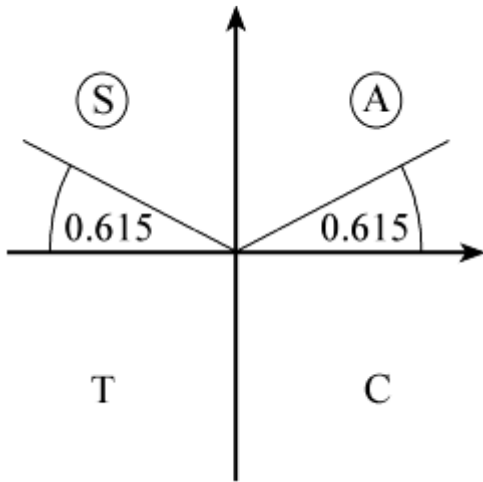
$$\text{So } \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

(d) Solve $\sin X = \frac{\sqrt{3}}{3}$ where $X = \theta + \frac{\pi}{3}$

Interval for X is $\frac{\pi}{3} \leq X \leq \frac{7\pi}{3}$ or 1.047 radians $\leq X \leq 7.33$ radians

First solution is $\sin^{-1} \left(\frac{\sqrt{3}}{3} \right) = 0.615$

As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.



$$X = \pi - 0.615, 2\pi + 0.615 = 2.526, 6.899$$

$$\text{So } \theta = X - \frac{\pi}{3} = 1.48, 5.85$$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise D, Question 1

Question:

Solve for θ , in the interval $0 \leq \theta \leq 360^\circ$, the following equations.
Give your answers to 3 significant figures where they are not exact.

(a) $4 \cos^2 \theta = 1$

(b) $2 \sin^2 \theta - 1 = 0$

(c) $3 \sin^2 \theta + \sin \theta = 0$

(d) $\tan^2 \theta - 2 \tan \theta - 10 = 0$

(e) $2 \cos^2 \theta - 5 \cos \theta + 2 = 0$

(f) $\sin^2 \theta - 2 \sin \theta - 1 = 0$

(g) $\tan^2 2\theta = 3$

(h) $4 \sin \theta = \tan \theta$

(i) $\sin \theta + 2 \cos^2 \theta + 1 = 0$

(j) $\tan^2 (\theta - 45^\circ) = 1$

(k) $3 \sin^2 \theta = \sin \theta \cos \theta$

(l) $4 \cos \theta (\cos \theta - 1) = -5 \cos \theta$

(m) $4 (\sin^2 \theta - \cos \theta) = 3 - 2 \cos \theta$

(n) $2 \sin^2 \theta = 3 (1 - \cos \theta)$

(o) $4 \cos^2 \theta - 5 \sin \theta - 5 = 0$

(p) $\cos^2 \frac{\theta}{2} = 1 + \sin \frac{\theta}{2}$

Solution:

(a) $4 \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = \frac{1}{4}$

So $\cos \theta = \pm \frac{1}{2}$

Solutions are $60^\circ, 120^\circ, 240^\circ, 300^\circ$

(b) $2 \sin^2 \theta - 1 = 0 \Rightarrow \sin^2 \theta = \frac{1}{2}$

$$\text{So } \sin \theta = \pm \frac{1}{\sqrt{2}}$$

Solutions are in all four quadrants at 45° to the horizontal.

$$\text{So } \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$\text{(c) Factorising, } \sin \theta (3 \sin \theta + 1) = 0$$

$$\text{So } \sin \theta = 0 \text{ or } \sin \theta = -\frac{1}{3}$$

Solutions of $\sin \theta = 0$ are $\theta = 0^\circ, 180^\circ, 360^\circ$ (from graph)

Solutions of $\sin \theta = -\frac{1}{3}$ are $\theta = 199^\circ, 341^\circ$ (3 s.f.) (3rd and 4th quadrants)

$$\text{(d) } \tan^2 \theta - 2 \tan \theta - 10 = 0$$

$$\text{So } \tan \theta = \frac{2 \pm \sqrt{4+40}}{2} = \frac{2 \pm \sqrt{44}}{2} (= -2.3166 \dots \text{ or } 4.3166 \dots)$$

Solutions of $\tan \theta = \frac{2 - \sqrt{44}}{2}$ are in the 2nd and 4th quadrants.

$$\text{So } \theta = 113.35^\circ, 293.3^\circ$$

Solutions of $\tan \theta = \frac{2 + \sqrt{44}}{2}$ are in the 1st and 3rd quadrants.

$$\text{So } \theta = 76.95 \dots^\circ, 256.95 \dots^\circ$$

$$\text{Solution set: } 77.0^\circ, 113^\circ, 257^\circ, 293^\circ$$

$$\text{(e) Factorise LHS of } 2 \cos^2 \theta - 5 \cos \theta + 2 = 0$$

$$(2 \cos \theta - 1)(\cos \theta - 2) = 0$$

$$\text{So } 2 \cos \theta - 1 = 0 \text{ or } \cos \theta - 2 = 0$$

As $\cos \theta \leq 1$, $\cos \theta = 2$ has no solutions.

Solutions of $\cos \theta = \frac{1}{2}$ are $\theta = 60^\circ, 300^\circ$

$$\text{(f) } \sin^2 \theta - 2 \sin \theta - 1 = 0$$

$$\text{So } \sin \theta = \frac{2 \pm \sqrt{8}}{2}$$

$$\text{Solve } \sin \theta = \frac{2 - \sqrt{8}}{2} \text{ as } \frac{2 + \sqrt{8}}{2} > 1$$

$$\theta = 204^\circ, 336^\circ \text{ (solutions are in 3rd and 4th quadrants as } \frac{2 - \sqrt{8}}{2} < 0)$$

$$\text{(g) } \tan^2 2\theta = 3 \Rightarrow \tan 2\theta = \pm \sqrt{3}$$

Solve $\tan X = +\sqrt{3}$ and $\tan X = -\sqrt{3}$, where $X = 2\theta$

Interval for X is $0 \leq X \leq 720^\circ$

For $\tan X = \sqrt{3}$, $X = 60^\circ, 240^\circ, 420^\circ, 600^\circ$

$$\text{So } \theta = \frac{X}{2} = 30^\circ, 120^\circ, 210^\circ, 300^\circ$$

For $\tan X = -\sqrt{3}$, $X = 120^\circ, 300^\circ, 480^\circ, 660^\circ$

$$\text{So } \theta = 60^\circ, 150^\circ, 240^\circ, 330^\circ$$

Solution set: $\theta = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$

$$\text{(h) } 4 \sin \theta = \tan \theta$$

$$\text{So } 4 \sin \theta = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow 4 \sin \theta \cos \theta = \sin \theta$$

$$\Rightarrow 4 \sin \theta \cos \theta - \sin \theta = 0$$

$$\Rightarrow \sin \theta (4 \cos \theta - 1) = 0$$

$$\text{So } \sin \theta = 0 \text{ or } \cos \theta = \frac{1}{4}$$

Solutions of $\sin \theta = 0$ are $0^\circ, 180^\circ, 360^\circ$

Solutions of $\cos \theta = \frac{1}{4}$ are $\cos^{-1} \left(\frac{1}{4} \right)$ and $360^\circ - \cos^{-1} \left(\frac{1}{4} \right)$

Solution set: $0^\circ, 75.5^\circ, 180^\circ, 284^\circ, 360^\circ$

(i) $\sin \theta + 2 \cos^2 \theta + 1 = 0$

So $\sin \theta + 2(1 - \sin^2 \theta) + 1 = 0$ using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$\Rightarrow 2 \sin^2 \theta - \sin \theta - 3 = 0$$

$$\Rightarrow (2 \sin \theta - 3)(\sin \theta + 1) = 0$$

So $\sin \theta = -1$ ($\sin \theta = \frac{3}{2}$ has no solution)

$$\Rightarrow \theta = 270^\circ$$

(j) $\tan^2 (\theta - 45^\circ) = 1$

So $\tan (\theta - 45^\circ) = 1$ or $\tan (\theta - 45^\circ) = -1$

So $\theta - 45^\circ = 45^\circ, 225^\circ$ (1st and 3rd quadrants)

or $\theta - 45^\circ = -45^\circ, 135^\circ, 315^\circ$ (2nd and 4th quadrants)

$$\Rightarrow \theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$$

(k) $3 \sin^2 \theta = \sin \theta \cos \theta$

$$\Rightarrow 3 \sin^2 \theta - \sin \theta \cos \theta = 0$$

$$\Rightarrow \sin \theta (3 \sin \theta - \cos \theta) = 0$$

So $\sin \theta = 0$ or $3 \sin \theta - \cos \theta = 0$

Solutions of $\sin \theta = 0$ are $\theta = 0^\circ, 180^\circ, 360^\circ$

For $3 \sin \theta - \cos \theta = 0$

$$3 \sin \theta = \cos \theta$$

$$\frac{3 \sin \theta}{3 \cos \theta} = \frac{\cos \theta}{3 \cos \theta}$$

$$\tan \theta = \frac{1}{3}$$

Solutions are $\theta = \tan^{-1} \left(\frac{1}{3} \right)$ and $180^\circ + \tan^{-1} \left(\frac{1}{3} \right) = 18.4^\circ, 198^\circ$

Solution set: $0^\circ, 18.4^\circ, 180^\circ, 198^\circ, 360^\circ$

(l) $4 \cos \theta (\cos \theta - 1) = -5 \cos \theta$

$$\Rightarrow \cos \theta [4(\cos \theta - 1) + 5] = 0$$

$$\Rightarrow \cos \theta (4 \cos \theta + 1) = 0$$

So $\cos \theta = 0$ or $\cos \theta = -\frac{1}{4}$

Solutions of $\cos \theta = 0$ are $90^\circ, 270^\circ$

Solutions of $\cos \theta = -\frac{1}{4}$ are $104^\circ, 256^\circ$ (3 s.f.) (2nd and 3rd quadrants)

Solution set: $90^\circ, 104^\circ, 256^\circ, 270^\circ$

(m) $4 \sin^2 \theta - 4 \cos \theta = 3 - 2 \cos \theta$

$$\Rightarrow 4(1 - \cos^2 \theta) - 4 \cos \theta = 3 - 2 \cos \theta$$

$$\Rightarrow 4 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

$$\text{So } \cos \theta = \frac{-2 \pm \sqrt{20}}{8} \left(= \frac{-1 \pm \sqrt{5}}{4} \right)$$

Solutions of $\cos \theta = \frac{-2 + \sqrt{20}}{8}$ are $72^\circ, 288^\circ$ (1st and 4th quadrants)

Solutions of $\cos \theta = \frac{-2 - \sqrt{20}}{8}$ are $144^\circ, 216^\circ$ (2nd and 3rd quadrants)

Solution set: $72.0^\circ, 144^\circ, 216^\circ, 288^\circ$

$$(n) 2 \sin^2 \theta = 3(1 - \cos \theta)$$

$$\Rightarrow 2(1 - \cos^2 \theta) = 3(1 - \cos \theta)$$

$$\Rightarrow 2(1 - \cos \theta)(1 + \cos \theta) = 3(1 - \cos \theta) \text{ or write as } a \cos^2 \theta + b \cos \theta + c \equiv 0$$

$$\Rightarrow (1 - \cos \theta)[2(1 + \cos \theta) - 3] = 0$$

$$\Rightarrow (1 - \cos \theta)(2 \cos \theta - 1) = 0$$

So $\cos \theta = 1$ or $\cos \theta = \frac{1}{2}$

Solutions are $0^\circ, 60^\circ, 300^\circ, 360^\circ$

$$(o) 4 \cos^2 \theta - 5 \sin \theta - 5 = 0$$

$$\Rightarrow 4(1 - \sin^2 \theta) - 5 \sin \theta - 5 = 0$$

$$\Rightarrow 4 \sin^2 \theta + 5 \sin \theta + 1 = 0$$

$$\Rightarrow (4 \sin \theta + 1)(\sin \theta + 1) = 0$$

So $\sin \theta = -1$ or $\sin \theta = -\frac{1}{4}$

Solution of $\sin \theta = -1$ is $\theta = 270^\circ$

Solutions of $\sin \theta = -\frac{1}{4}$ are $\theta = 194^\circ, 346^\circ$ (3 s.f.) (3rd and 4th quadrants)

Solution set: $194^\circ, 270^\circ, 346^\circ$

$$(p) \cos^2 \frac{\theta}{2} = 1 + \sin \frac{\theta}{2}$$

$$\Rightarrow 1 - \sin^2 \frac{\theta}{2} = 1 + \sin \frac{\theta}{2}$$

$$\Rightarrow \sin^2 \frac{\theta}{2} + \sin \frac{\theta}{2} = 0$$

$$\Rightarrow \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} + 1 \right) = 0$$

So $\sin \frac{\theta}{2} = 0$ or $\sin \frac{\theta}{2} = -1$

Solve $\sin X = 0$ and $\sin X = -1$ where $X = \frac{\theta}{2}$

Interval for X is $0 \leq X \leq 180^\circ$

$X = 0^\circ, 180^\circ$ ($\sin X = -1$ has no solutions in the interval)

So $\theta = 2X = 0^\circ, 360^\circ$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise D, Question 2

Question:

Solve for θ , in the interval $-180^\circ \leq \theta \leq 180^\circ$, the following equations.
Give your answers to 3 significant figures where they are not exact.

(a) $\sin^2 2\theta = 1$

(b) $\tan^2 \theta = 2 \tan \theta$

(c) $\cos \theta (\cos \theta - 2) = 1$

(d) $\sin^2 (\theta + 10^\circ) = 0.8$

(e) $\cos^2 3\theta - \cos 3\theta = 2$

(f) $5 \sin^2 \theta = 4 \cos^2 \theta$

(g) $\tan \theta = \cos \theta$

(h) $2 \sin^2 \theta + 3 \cos \theta = 1$

Solution:

(a) Solve $\sin^2 X = 1$ where $X = 2\theta$

Interval for X is $-360^\circ \leq X \leq 360^\circ$

$\sin X = +1$ gives $X = -270^\circ, 90^\circ$

$\sin X = -1$ gives $X = -90^\circ, +270^\circ$

$X = -270^\circ, -90^\circ, +90^\circ, +270^\circ$

So $\theta = \frac{X}{2} = -135^\circ, -45^\circ, +45^\circ, +135^\circ$

(b) $\tan^2 \theta = 2 \tan \theta$

$\Rightarrow \tan^2 \theta - 2 \tan \theta = 0$

$\Rightarrow \tan \theta (\tan \theta - 2) = 0$

So $\tan \theta = 0$ or $\tan \theta = 2$ (1st and 3rd quadrants)

Solutions are $(-180^\circ, 0^\circ, 180^\circ), (-116.6^\circ, 63.4^\circ)$

Solution set: $-180^\circ, -117^\circ, 0^\circ, 63.4^\circ, 180^\circ$

(c) $\cos^2 \theta - 2 \cos \theta = 1$

$\Rightarrow \cos^2 \theta - 2 \cos \theta - 1 = 0$

So $\cos \theta = \frac{2 \pm \sqrt{8}}{2}$

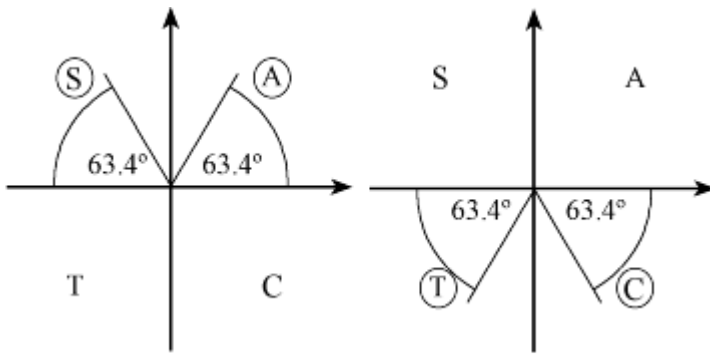
$\Rightarrow \cos \theta = \frac{2 - \sqrt{8}}{2}$ (as $\frac{2 + \sqrt{8}}{2} > 1$)

Solutions are $\pm 114^\circ$ (2nd and 3rd quadrants)

(d) $\sin^2 (\theta + 10^\circ) = 0.8$

$\Rightarrow \sin (\theta + 10^\circ) = +\sqrt{0.8}$ or $\sin (\theta + 10^\circ) = -\sqrt{0.8}$

Either $(\theta + 10^\circ) = 63.4^\circ, 116.6^\circ$ or $(\theta + 10^\circ) = -116.6^\circ, -63.4^\circ$



So $\theta = -127^\circ, -73.4^\circ, 53.4^\circ, 107^\circ$ (3 s.f.)

$$(e) \cos^2 3\theta - \cos 3\theta - 2 = 0$$

$$(\cos 3\theta - 2)(\cos 3\theta + 1) = 0$$

$$\text{So } \cos 3\theta = -1 \quad (\cos 3\theta \neq 2)$$

Solve $\cos X = -1$ where $X = 3\theta$

$$\text{Interval for } X \text{ is } -540^\circ \leq X \leq 540^\circ$$

From the graph of $y = \cos X$, $\cos X = -1$ where
 $X = -540^\circ, -180^\circ, 180^\circ, 540^\circ$

$$\text{So } \theta = \frac{X}{3} = -180^\circ, -60^\circ, +60^\circ, +180^\circ$$

$$(f) 5 \sin^2 \theta = 4 \cos^2 \theta$$

$$\Rightarrow \tan^2 \theta = \frac{4}{5} \text{ as } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\text{So } \tan \theta = \pm \sqrt{\frac{4}{5}}$$

There are solutions from each of the quadrants (angle to horizontal is 41.8°)
 $\theta = \pm 138^\circ, \pm 41.8^\circ$

$$(g) \tan \theta = \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \cos \theta$$

$$\Rightarrow \sin \theta = \cos^2 \theta$$

$$\Rightarrow \sin \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta + \sin \theta - 1 = 0$$

$$\text{So } \sin \theta = \frac{-1 \pm \sqrt{5}}{2}$$

$$\text{Only solutions from } \sin \theta = \frac{-1 + \sqrt{5}}{2} \text{ (as } \frac{-1 - \sqrt{5}}{2} < -1)$$

Solutions are $\theta = 38.2^\circ, 142^\circ$ (1st and 2nd quadrants)

$$(h) 2 \sin^2 \theta + 3 \cos \theta = 1$$

$$\Rightarrow 2(1 - \cos^2 \theta) + 3 \cos \theta = 1$$

$$\Rightarrow 2 \cos^2 \theta - 3 \cos \theta - 1 = 0$$

$$\text{So } \cos \theta = \frac{3 \pm \sqrt{17}}{4}$$

$$\text{Only solutions of } \cos \theta = \frac{3 - \sqrt{17}}{4} \text{ (as } \frac{3 + \sqrt{17}}{4} > 1)$$

Solutions are $\theta = \pm 106^\circ$ (2nd and 3rd quadrants)

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise D, Question 3

Question:

Solve for x , in the interval $0 \leq x \leq 2\pi$, the following equations.

Give your answers to 3 significant figures unless they can be written in the form $\frac{a}{b}\pi$, where a and b are integers.

(a) $\tan^2 \frac{1}{2}x = 1$

(b) $2 \sin^2 \left(x + \frac{\pi}{3} \right) = 1$

(c) $3 \tan x = 2 \tan^2 x$

(d) $\sin^2 x + 2 \sin x \cos x = 0$

(e) $6 \sin^2 x + \cos x - 4 = 0$

(f) $\cos^2 x - 6 \sin x = 5$

(g) $2 \sin^2 x = 3 \sin x \cos x + 2 \cos^2 x$

Solution:

(a) $\tan^2 \frac{1}{2}x = 1$

$$\Rightarrow \tan \frac{1}{2}x = \pm 1$$

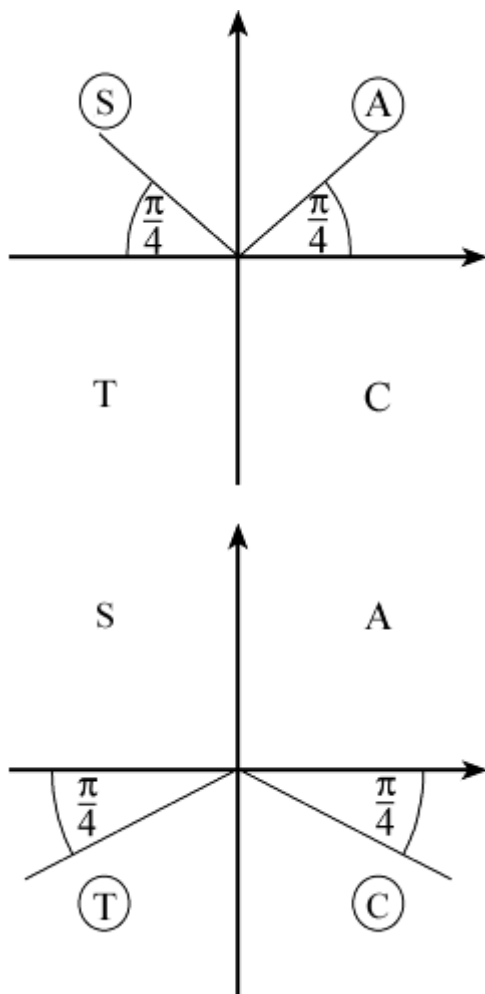
$$\Rightarrow \frac{1}{2}x = \frac{\pi}{4}, \frac{3\pi}{4} \quad \left(0 \leq \frac{1}{2}x \leq \pi \right)$$

So $x = \frac{\pi}{2}, \frac{3\pi}{2}$

(b) $2 \sin^2 \left(x + \frac{\pi}{3} \right) = 1$ for $\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{7\pi}{3}$

$$\Rightarrow \sin^2 \left(x + \frac{\pi}{3} \right) = \frac{1}{2}$$

So $\sin \left(x + \frac{\pi}{3} \right) = \frac{1}{\sqrt{2}}$ or $\sin \left(x + \frac{\pi}{3} \right) = -\frac{1}{\sqrt{2}}$



$$x + \frac{\pi}{3} = \frac{3\pi}{4}, \frac{9\pi}{4} \text{ or } x + \frac{\pi}{3} = +\frac{5\pi}{4}, +\frac{7\pi}{4}$$

$$\text{So } x = \frac{3\pi}{4} - \frac{\pi}{3}, \frac{9\pi}{4} - \frac{\pi}{3} \text{ or } x = \frac{5\pi}{4} - \frac{\pi}{3}, \frac{7\pi}{4} - \frac{\pi}{3}$$

$$\text{Solutions are } x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$$

$$\begin{aligned} \text{(c) } 3 \tan x &= 2 \tan^2 x \\ \Rightarrow 2 \tan^2 x - 3 \tan x &= 0 \\ \Rightarrow \tan x (2 \tan x - 3) &= 0 \end{aligned}$$

$$\text{So } \tan x = 0 \text{ or } \tan x = \frac{3}{2}$$

$$x = (0, \pi, 2\pi), (0.983, \pi + 0.983) = 0, 0.983, \pi, 4.12, 2\pi$$

$$\begin{aligned} \text{(d) } \sin^2 x + 2 \sin x \cos x &= 0 \\ \Rightarrow \sin x (\sin x + 2 \cos x) &= 0 \end{aligned}$$

$$\text{So } \sin x = 0 \text{ or } \sin x + 2 \cos x = 0$$

$$\sin x = 0 \text{ gives } x = 0, \pi, 2\pi$$

$$\sin x + 2 \cos x = 0 \Rightarrow \tan x = -2$$

$$\text{Solutions are } 2.03, 5.18 \text{ radians (2nd and 4th quadrants)}$$

$$\text{Solution set: } 0, 2.03, \pi, 5.18, 2\pi$$

$$\begin{aligned} \text{(e) } 6 \sin^2 x + \cos x - 4 &= 0 \\ \Rightarrow 6(1 - \cos^2 x) + \cos x - 4 &= 0 \\ \Rightarrow 6 \cos^2 x - \cos x - 2 &= 0 \end{aligned}$$

$$\Rightarrow (3 \cos x - 2)(2 \cos x + 1) = 0$$

$$\text{So } \cos x = +\frac{2}{3} \text{ or } \cos x = -\frac{1}{2}$$

$$\text{Solutions of } \cos x = +\frac{2}{3} \text{ are } \cos^{-1}\left(\frac{2}{3}\right), 2\pi - \cos^{-1}\left(\frac{2}{3}\right) = 0.841, 5.44$$

$$\text{Solutions of } \cos x = -\frac{1}{2} \text{ are } \cos^{-1}\left(-\frac{1}{2}\right), 2\pi - \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\text{Solutions are } 0.841, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.44$$

$$(f) \cos^2 x - 6 \sin x = 5$$

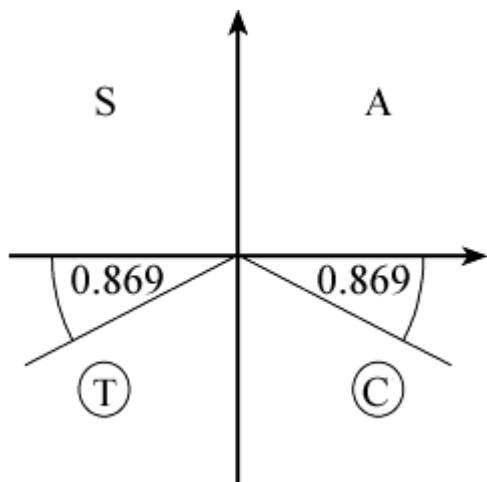
$$\Rightarrow (1 - \sin^2 x) - 6 \sin x = 5$$

$$\Rightarrow \sin^2 x + 6 \sin x + 4 = 0$$

$$\text{So } \sin x = \frac{-6 \pm \sqrt{20}}{2} \quad \left(= -3 \pm \sqrt{5} \right)$$

$$\text{As } \frac{-6 - \sqrt{20}}{2} < -1, \text{ there are no solutions of } \sin x = \frac{-6 - \sqrt{20}}{2}$$

$$\text{Consider solutions of } \sin x = \frac{-6 + \sqrt{20}}{2}$$



$$\sin^{-1}\left(\frac{-6 + \sqrt{20}}{2}\right) = -0.869 \text{ (not in given interval)}$$

$$\text{Solutions are } \pi + 0.869, 2\pi - 0.869 = 4.01, 5.41$$

$$(g) 2 \sin^2 x - 3 \sin x \cos x - 2 \cos^2 x = 0$$

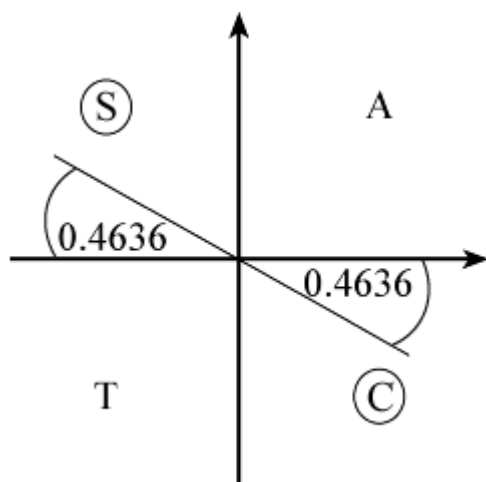
$$\Rightarrow (2 \sin x + \cos x)(\sin x - 2 \cos x) = 0$$

$$\Rightarrow 2 \sin x + \cos x = 0 \text{ or } \sin x - 2 \cos x = 0$$

$$\text{So } \tan x = -\frac{1}{2} \text{ or } \tan x = 2$$

$$\text{Consider solutions of } \tan x = -\frac{1}{2}$$

$$\text{First solution is } \tan^{-1}\left(-\frac{1}{2}\right) = -0.4636 \dots \text{ (not in interval)}$$



Solutions are $\pi - 0.4636$, $2\pi - 0.4636 = 2.68, 5.82$

Solutions of $\tan x = 2$ are $\tan^{-1} 2$, $\pi + \tan^{-1} 2 = 1.11, 4.25$

Solution set: $x = 1.11, 2.68, 4.25, 5.82$ (3 s.f.)

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise E, Question 1

Question:

Given that angle A is obtuse and $\cos A = -\sqrt{\frac{7}{11}}$, show that $\tan A = \frac{-2\sqrt{7}}{7}$.

Solution:

Using $\sin^2 A + \cos^2 A \equiv 1$

$$\sin^2 A + \left(-\sqrt{\frac{7}{11}}\right)^2 = 1$$

$$\sin^2 A = 1 - \frac{7}{11} = \frac{4}{11}$$

$$\sin A = \pm \frac{2}{\sqrt{11}}$$

But A is in the second quadrant (obtuse), so $\sin A$ is +ve.

$$\text{So } \sin A = +\frac{2}{\sqrt{11}}$$

$$\text{Using } \tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{\left(\frac{2}{\sqrt{11}}\right)}{-\sqrt{\frac{7}{11}}} = -\frac{2}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{7}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7} \text{ (rationalising the denominator)}$$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

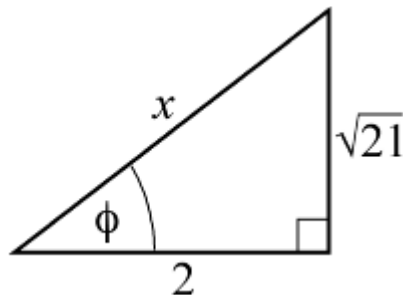
Exercise E, Question 2

Question:

Given that angle B is reflex and $\tan B = + \frac{\sqrt{21}}{2}$, find the exact value of: (a) $\sin B$, (b) $\cos B$.

Solution:

Draw a right-angled triangle with an angle ϕ where $\tan \phi = + \frac{\sqrt{21}}{2}$.



Using Pythagoras' Theorem to find the hypotenuse:

$$x^2 = 2^2 + (\sqrt{21})^2 = 4 + 21 = 25$$

So $x = 5$

$$(a) \sin \phi = \frac{\sqrt{21}}{5}$$

As B is reflex and $\tan B$ is +ve, B is in the third quadrant.

$$\text{So } \sin B = -\sin \phi = -\frac{\sqrt{21}}{5}$$

$$(b) \text{ From the diagram } \cos \phi = \frac{2}{5}$$

$$B \text{ is in the third quadrant, so } \cos B = -\cos \phi = -\frac{2}{5}$$

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Edexcel Modular Mathematics for AS and A-Level

Trigonometrical identities and simple equations

Exercise E, Question 3

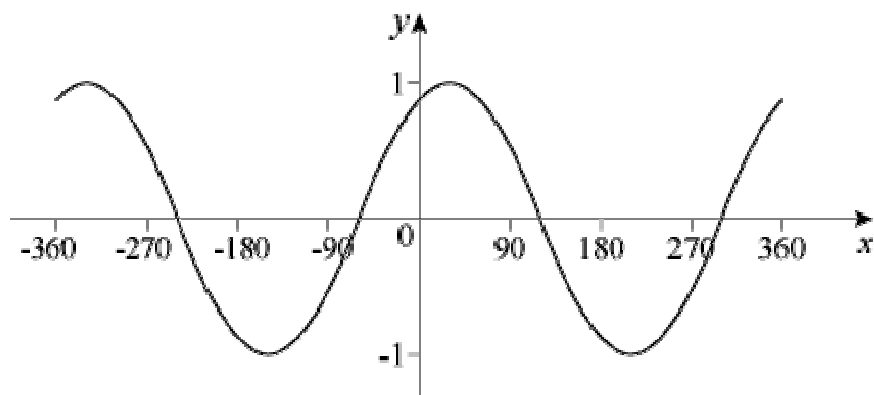
Question:

(a) Sketch the graph of $y = \sin (x + 60)^\circ$, in the interval $-360 \leq x \leq 360$, giving the coordinates of points of intersection with the axes.

(b) Calculate the values of the x -coordinates of the points in which the line $y = \frac{1}{2}$ intersects the curve.

Solution:

(a) The graph of $y = \sin (x + 60)^\circ$ is the graph of $y = \sin x^\circ$ translated by 60 to the left.



The curve meets the x -axis at

$(-240, 0)$, $(-60, 0)$, $(120, 0)$ and $(300, 0)$

The curve meets the y -axis, where $x = 0$.

$$\text{So } y = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

Coordinates are $\left(0, \frac{\sqrt{3}}{2}\right)$

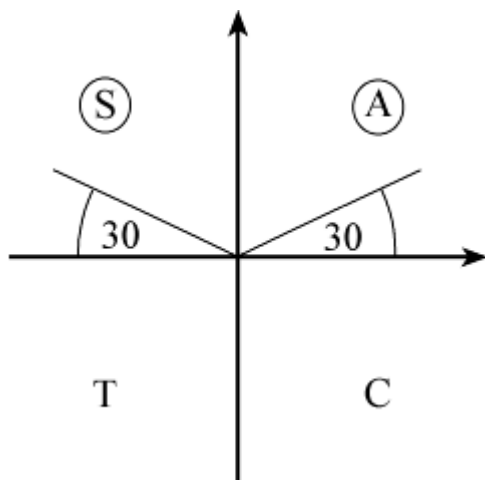
(b) The line meets the curve where $\sin (x + 60)^\circ = \frac{1}{2}$

Let $(x + 60) = X$ and solve $\sin X^\circ = \frac{1}{2}$ where $-300 \leq X \leq 420$

$$\sin X^\circ = \frac{1}{2}$$

First solution is $X = 30$ (your calculator solution)

As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.



Read off all solutions in the interval $-300 \leq X \leq 420$

$$X = -210, 30, 150, 390$$

$$x + 60 = -210, 30, 150, 390$$

$$\text{So } x = -270, -30, 90, 330$$

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Trigonometrical identities and simple equations

Exercise E, Question 4

Question:

Simplify the following expressions:

(a) $\cos^4 \theta - \sin^4 \theta$

(b) $\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$

(c) $\cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta$

Solution:

(a) Factorise $\cos^4 \theta - \sin^4 \theta$ (difference of two squares)

$$\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta + \sin^2 \theta) (\cos^2 \theta - \sin^2 \theta) = (1) (\cos^2 \theta - \sin^2 \theta) \text{ (as } \sin^2 \theta + \cos^2 \theta \equiv 1)$$

$$\text{So } \cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

(b) Factorise $\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$

$$\sin^2 3\theta - \sin^2 3\theta \cos^2 3\theta$$

$$= \sin^2 3\theta (1 - \cos^2 3\theta) \text{ use } \sin^2 3\theta + \cos^2 3\theta \equiv 1$$

$$= \sin^2 3\theta (\sin^2 3\theta)$$

$$= \sin^4 3\theta$$

$$(c) \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta = (\cos^2 \theta + \sin^2 \theta)^2 = 1$$

$$\text{since } \sin^2 \theta + \cos^2 \theta \equiv 1$$

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Trigonometrical identities and simple equations

Exercise E, Question 5

Question:

- (a) Given that $2(\sin x + 2 \cos x) = \sin x + 5 \cos x$, find the exact value of $\tan x$.
- (b) Given that $\sin x \cos y + 3 \cos x \sin y = 2 \sin x \sin y - 4 \cos x \cos y$, express $\tan y$ in terms of $\tan x$.

Solution:

$$\begin{aligned} \text{(a)} \quad & 2(\sin x + 2 \cos x) = \sin x + 5 \cos x \\ \Rightarrow & 2 \sin x + 4 \cos x = \sin x + 5 \cos x \\ \Rightarrow & 2 \sin x - \sin x = 5 \cos x - 4 \cos x \\ \Rightarrow & \sin x = \cos x \text{ divide both sides by } \cos x \end{aligned}$$

$$\text{So } \tan x = 1$$

$$\begin{aligned} \text{(b)} \quad & \sin x \cos y + 3 \cos x \sin y = 2 \sin x \sin y - 4 \cos x \cos y \\ \Rightarrow & \frac{\sin x \cos y}{\cos x \cos y} + \frac{3 \cos x \sin y}{\cos x \cos y} = \frac{2 \sin x \sin y}{\cos x \cos y} - \frac{4 \cos x \cos y}{\cos x \cos y} \end{aligned}$$

$$\Rightarrow \tan x + 3 \tan y = 2 \tan x \tan y - 4$$

$$\Rightarrow 2 \tan x \tan y - 3 \tan y = 4 + \tan x$$

$$\Rightarrow \tan y (2 \tan x - 3) = 4 + \tan x$$

$$\text{So } \tan y = \frac{4 + \tan x}{2 \tan x - 3}$$

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Trigonometrical identities and simple equations

Exercise E, Question 6

Question:

Show that, for all values of θ :

(a) $(1 + \sin \theta)^2 + \cos^2 \theta = 2(1 + \sin \theta)$

(b) $\cos^4 \theta + \sin^2 \theta = \sin^4 \theta + \cos^2 \theta$

Solution:

(a) LHS = $(1 + 2 \sin \theta + \sin^2 \theta) + \cos^2 \theta$
= $1 + 2 \sin \theta + 1$ since $\sin^2 \theta + \cos^2 \theta \equiv 1$
= $2 + 2 \sin \theta$
= $2(1 + \sin \theta)$
= RHS

(b) LHS = $\cos^4 \theta + \sin^2 \theta$
= $(\cos^2 \theta)^2 + \sin^2 \theta$
= $(1 - \sin^2 \theta)^2 + \sin^2 \theta$ since $\sin^2 \theta + \cos^2 \theta \equiv 1$
= $1 - 2 \sin^2 \theta + \sin^4 \theta + \sin^2 \theta$
= $(1 - \sin^2 \theta) + \sin^4 \theta$
= $\cos^2 \theta + \sin^4 \theta$ using $\sin^2 \theta + \cos^2 \theta \equiv 1$
= RHS

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Trigonometrical identities and simple equations

Exercise E, Question 7

Question:

Without attempting to solve them, state how many solutions the following equations have in the interval $0 \leq \theta \leq 360^\circ$. Give a brief reason for your answer.

(a) $2 \sin \theta = 3$

(b) $\sin \theta = -\cos \theta$

(c) $2 \sin \theta + 3 \cos \theta + 6 = 0$

(d) $\tan \theta + \frac{1}{\tan \theta} = 0$

Solution:

(a) $\sin \theta = \frac{3}{2}$ has no solutions as $-1 \leq \sin \theta \leq 1$

(b) $\sin \theta = -\cos \theta$
 $\Rightarrow \tan \theta = -1$

Look at graph of $y = \tan \theta$ in the interval $0 \leq \theta \leq 360^\circ$.
 There are 2 solutions

(c) The minimum value of $2 \sin \theta$ is -2
 The minimum value of $3 \cos \theta$ is -3
 Each minimum value is for a different θ .
 So the minimum value of $2 \sin \theta + 3 \cos \theta > -5$.
 There are no solutions of $2 \sin \theta + 3 \cos \theta + 6 = 0$ as the LHS can never be zero.

(d) Solving $\tan \theta + \frac{1}{\tan \theta} = 0$ is equivalent to solving $\tan^2 \theta = -1$, which has no real solutions, so there are no solutions.

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Trigonometrical identities and simple equations

Exercise E, Question 8

Question:

- (a) Factorise $4xy - y^2 + 4x - y$.
- (b) Solve the equation $4 \sin \theta \cos \theta - \cos^2 \theta + 4 \sin \theta - \cos \theta = 0$, in the interval $0 \leq \theta \leq 360^\circ$.

Solution:

(a) $4xy - y^2 + 4x - y \equiv y(4x - y) + (4x - y) = (4x - y)(y + 1)$

(b) Using (a) with $x = \sin \theta$, $y = \cos \theta$
 $4 \sin \theta \cos \theta - \cos^2 \theta + 4 \sin \theta - \cos \theta = 0$
 $\Rightarrow (4 \sin \theta - \cos \theta)(\cos \theta + 1) = 0$

So $4 \sin \theta - \cos \theta = 0$ or $\cos \theta + 1 = 0$

$4 \sin \theta - \cos \theta = 0 \Rightarrow \tan \theta = \frac{1}{4}$

Calculator solution is $\theta = 14.0^\circ$

$\tan \theta$ is +ve so θ is in the 1st and 3rd quadrants

So $\theta = 14.0^\circ, 194^\circ$

$\cos \theta + 1 = 0 \Rightarrow \cos \theta = -1$

So $\theta = +180^\circ$ (from graph)

Solutions are $\theta = 14.0^\circ, 180^\circ, 194^\circ$

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Trigonometrical identities and simple equations

Exercise E, Question 9

Question:

(a) Express $4 \cos 3\theta^\circ - \sin (90 - 3\theta)^\circ$ as a single trigonometric function.

(b) Hence solve $4 \cos 3\theta^\circ - \sin (90 - 3\theta)^\circ = 2$ in the interval $0 \leq \theta \leq 360$. Give your answers to 3 significant figures.

Solution:

(a) As $\sin (90 - \theta)^\circ \equiv \cos \theta^\circ$, $\sin (90 - 3\theta)^\circ \equiv \cos 3\theta^\circ$
 So $4 \cos 3\theta^\circ - \sin (90 - 3\theta)^\circ = 4 \cos 3\theta^\circ - \cos 3\theta^\circ = 3 \cos 3\theta^\circ$

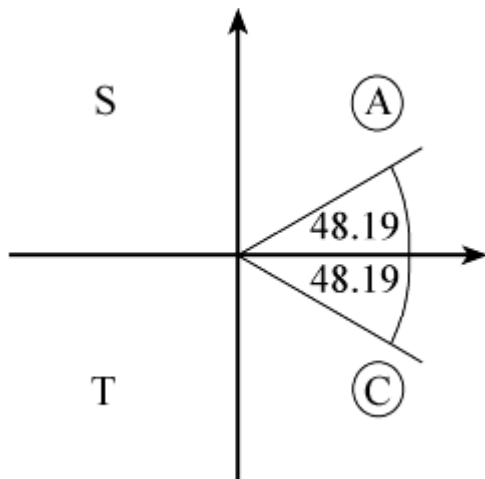
(b) Using (a) $4 \cos 3\theta^\circ - \sin (90 - 3\theta)^\circ = 2$
 is equivalent to $3 \cos 3\theta^\circ = 2$

$$\text{so } \cos 3\theta^\circ = \frac{2}{3}$$

Let $X = 3\theta$ and solve $\cos X^\circ = \frac{2}{3}$ in the interval $0 \leq X \leq 1080$

The calculator solution is $X = 48.19$

As $\cos X^\circ$ is +ve, X is in the 1st and 4th quadrant.



Read off all solutions in the interval $0 \leq X \leq 1080$

$$X = 48.19, 311.81, 408.19, 671.81, 768.19, 1031.81$$

$$\text{So } \theta = \frac{1}{3}X = 16.1, 104, 136, 224, 256, 344 \text{ (3 s.f.)}$$

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Trigonometrical identities and simple equations

Exercise E, Question 10

Question:

Find, in radians to two decimal places, the value of x in the interval $0 \leq x \leq 2\pi$, for which $3 \sin^2 x + \sin x - 2 = 0$. [E]

Solution:

$$3 \sin^2 x + \sin x - 2 = 0$$
$$(3 \sin x - 2)(\sin x + 1) = 0 \text{ factorising}$$

$$\text{So } \sin x = \frac{2}{3} \text{ or } \sin x = -1$$

For $\sin x = \frac{2}{3}$ your calculator answer is 0.73 (2 d.p.)

As $\sin x$ is +ve, x is in the 1st and 2nd quadrants.
So second solution is $(\pi - 0.73) = 2.41$ (2 d.p.)

For $\sin x = -1$, $x = \frac{3\pi}{2} = 4.71$ (2 d.p.)

So $x = 0.73, 2.41, 4.71$

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Trigonometrical identities and simple equations

Exercise E, Question 11

Question:

Given that $2 \sin 2\theta = \cos 2\theta$:

(a) Show that $\tan 2\theta = 0.5$.

(b) Hence find the value of θ , to one decimal place, in the interval $0 \leq \theta < 360^\circ$ for which $2 \sin 2\theta = \cos 2\theta$. **[E]**

Solution:

(a) $2 \sin 2\theta = \cos 2\theta$

$$\Rightarrow \frac{2 \sin 2\theta}{\cos 2\theta} = 1$$

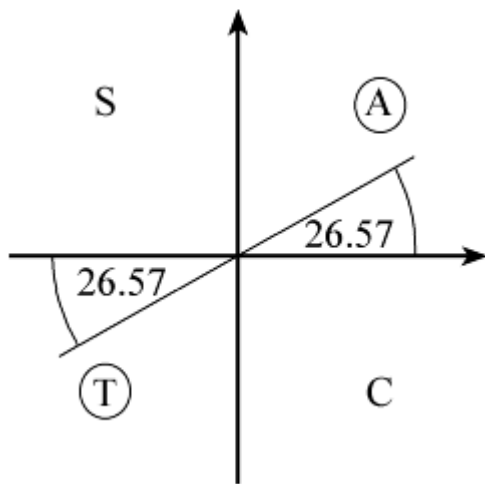
$$\Rightarrow 2 \tan 2\theta = 1 \quad \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$$

So $\tan 2\theta = 0.5$

(b) Solve $\tan 2\theta = 0.5$ in the interval $0 \leq \theta < 360$
or $\tan X = 0.5$ where $X = 2\theta$, $0 \leq X < 720$

The calculator solution for $\tan^{-1} 0.5 = 26.57$

As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



Read off solutions for X in the interval $0 \leq X < 720$

$X = 26.57, 206.57, 386.57, 566.57$

$X = 2\theta$

So $\theta = \frac{1}{2}X = 13.3, 103.3, 193.3, 283.3$ (1 d.p.)

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Trigonometrical identities and simple equations

Exercise E, Question 12

Question:

Find all the values of θ in the interval $0 \leq \theta < 360$ for which:

(a) $\cos (\theta + 75)^\circ = 0.5$.

(b) $\sin 2\theta^\circ = 0.7$, giving your answers to one decimal place. [E]

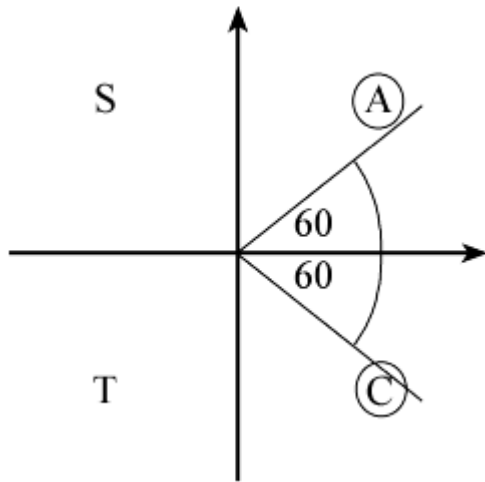
Solution:

(a) $\cos (\theta + 75)^\circ = 0.5$

Solve $\cos X^\circ = 0.5$ where $X = \theta + 75$, $75 \leq X < 435$

Your calculator solution for X is 60

As $\cos X$ is +ve, X is in the 1st and 4th quadrants.



Read off all solutions in the interval $75 \leq X < 435$

$$X = 300, 420$$

$$\theta + 75 = 300, 420$$

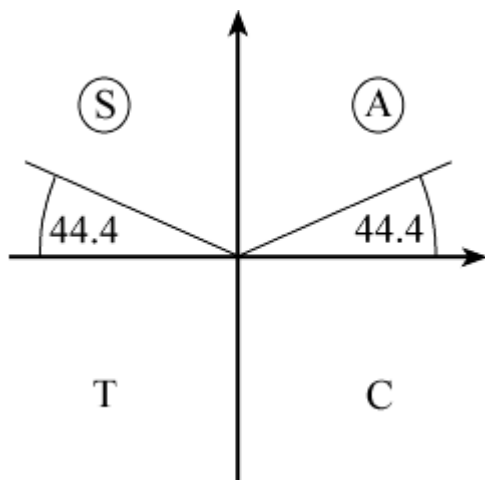
$$\text{So } \theta = 225, 345$$

(b) $\sin 2\theta^\circ = 0.7$ in the interval $0 \leq \theta < 360$

Solve $\sin X^\circ = 0.7$ where $X = 2\theta$, $0 \leq X < 720$

Your calculator solution is 44.4

As $\sin X$ is +ve, X is in the 1st and 2nd quadrants.



Read off solutions in the interval $0 \leq X < 720$

$$X = 44.4, 135.6, 404.4, 495.6$$

$$X = 2\theta$$

$$\text{So } \theta = \frac{1}{2}X = 22.2, 67.8, 202.2, 247.8 \text{ (1 d.p.)}$$

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Exercise E, Question 13

Question:

(a) Find the coordinates of the point where the graph of $y = 2 \sin \left(2x + \frac{5}{6}\pi \right)$ crosses the y-axis.

(b) Find the values of x , where $0 \leq x \leq 2\pi$, for which $y = \sqrt{2}$. **[E]**

Solution:

(a) $y = 2 \sin \left(2x + \frac{5}{6}\pi \right)$ crosses the y-axis where $x = 0$

$$\text{So } y = 2 \sin \frac{5}{6}\pi = 2 \times \frac{1}{2} = 1$$

Coordinates are (0, 1)

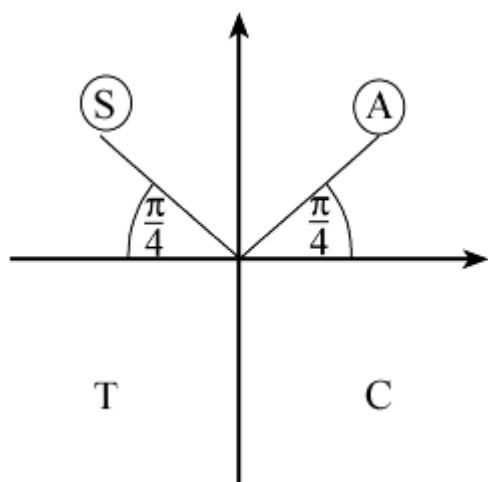
(b) Solve $2 \sin \left(2x + \frac{5}{6}\pi \right) = \sqrt{2}$ in the interval $0 \leq x \leq 2\pi$

$$\text{So } \sin \left(2x + \frac{5}{6}\pi \right) = \frac{\sqrt{2}}{2}$$

$$\text{or } \sin X = \frac{\sqrt{2}}{2} \text{ where } \frac{5}{6}\pi \leq X \leq 4\frac{5}{6}\pi$$

Your calculator solution is $\frac{\pi}{4}$

As $\sin X$ is +ve, X lies in the 1st and 2nd quadrants.



Read off solutions for X in the interval $\frac{5}{6}\pi \leq X \leq 4\frac{5}{6}\pi$

(Note: first value of X in interval is on second revolution.)

$$X = \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$$

$$2x + \frac{5}{6}\pi = \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$$

$$2x = \frac{9\pi}{4} - \frac{5\pi}{6}, \frac{11\pi}{4} - \frac{5\pi}{6}, \frac{17\pi}{4} - \frac{5\pi}{6}, \frac{19\pi}{4} - \frac{5\pi}{6}$$

$$2x = \frac{17\pi}{12}, \frac{23\pi}{12}, \frac{41\pi}{12}, \frac{47\pi}{12}$$

$$\text{So } x = \frac{17\pi}{24}, \frac{23\pi}{24}, \frac{41\pi}{24}, \frac{47\pi}{24}$$

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Trigonometrical identities and simple equations

Exercise E, Question 14

Question:

Find, giving your answers in terms of π , all values of θ in the interval $0 < \theta < 2\pi$, for which:

(a) $\tan \left(\theta + \frac{\pi}{3} \right) = 1$

(b) $\sin 2\theta = -\frac{\sqrt{3}}{2}$ [E]

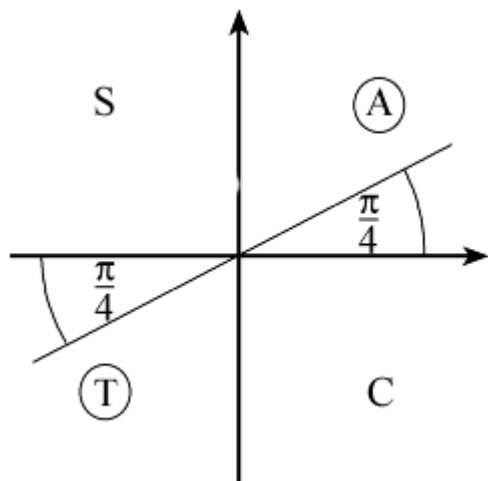
Solution:

(a) $\tan \left(\theta + \frac{\pi}{3} \right) = 1$ in the interval $0 < \theta < 2\pi$

Solve $\tan X = 1$ where $\frac{\pi}{3} < X < \frac{7\pi}{3}$

Calculator solution is $\frac{\pi}{4}$

As $\tan X$ is +ve, X is in the 1st and 3rd quadrants.



Read off solutions for X in the interval $\frac{\pi}{3} < X < \frac{7\pi}{3}$

$$X = \frac{5\pi}{4}, \frac{9\pi}{4}$$

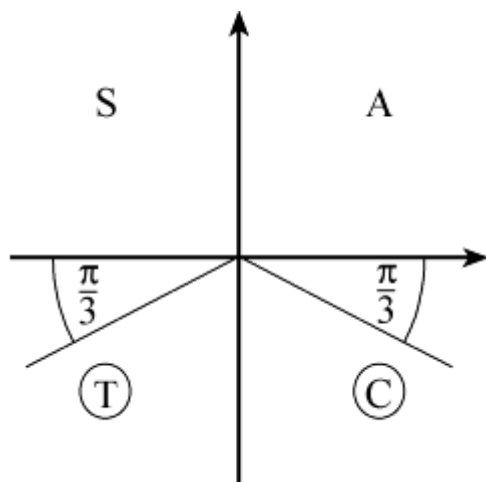
$$\theta + \frac{\pi}{3} = \frac{5\pi}{4}, \frac{9\pi}{4}$$

$$\text{So } \theta = \frac{5\pi}{4} - \frac{\pi}{3}, \frac{9\pi}{4} - \frac{\pi}{3} = \frac{11\pi}{12}, \frac{23\pi}{12}$$

(b) Solve $\sin X = -\frac{\sqrt{3}}{2}$ where $X = 2\theta$, $0 < \theta < 4\pi$

Calculator answer is $-\frac{\pi}{3}$

As $\sin X$ is $-ve$, X is in the 3rd and 4th quadrants.



Read off solutions for X in the interval $0 < \theta < 4\pi$

$$X = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$$

$$\text{So } \theta = \frac{1}{2}X = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$$

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Trigonometrical identities and simple equations

Exercise E, Question 15

Question:

Find the values of x in the interval $0 < x < 270^\circ$ which satisfy the equation

$$\frac{\cos 2x + 0.5}{1 - \cos 2x} = 2$$

Solution:

Multiply both sides of equation by $(1 - \cos 2x)$ (providing $\cos 2x \neq 1$)

(Note: In the interval given $\cos 2x$ is never equal to 1.)

$$\text{So } \cos 2x + 0.5 = 2 - 2 \cos 2x$$

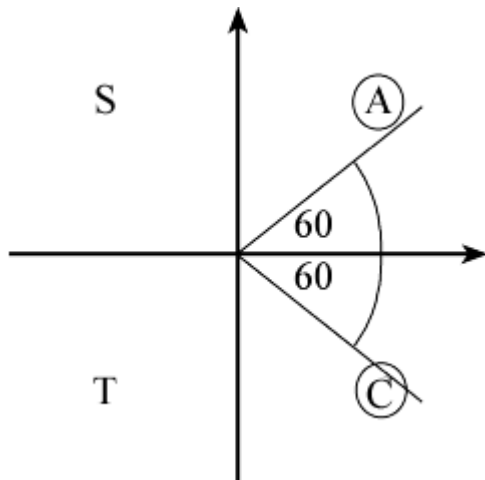
$$\Rightarrow 3 \cos 2x = \frac{3}{2}$$

$$\text{So } \cos 2x = \frac{1}{2}$$

Solve $\cos X = \frac{1}{2}$ where $X = 2x$, $0 < X < 540$

Calculator solution is 60°

As $\cos X$ is +ve, X is in 1st and 4th quadrants.



Read off solutions for X in the interval $0 < X < 540$

$$X = 60^\circ, 300^\circ, 420^\circ$$

$$\text{So } x = \frac{1}{2}X = 30^\circ, 150^\circ, 210^\circ$$

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Trigonometrical identities and simple equations

Exercise E, Question 16

Question:

Find, to the nearest integer, the values of x in the interval $0 \leq x < 180^\circ$ for which $3 \sin^2 3x - 7 \cos 3x - 5 = 0$.

[E]

Solution:

Using $\sin^2 3x + \cos^2 3x \equiv 1$

$$3(1 - \cos^2 3x) - 7 \cos 3x - 5 = 0$$

$$\Rightarrow 3 \cos^2 3x + 7 \cos 3x + 2 = 0$$

$$\Rightarrow (3 \cos 3x + 1)(\cos 3x + 2) = 0 \text{ factorising}$$

So $3 \cos 3x + 1 = 0$ or $\cos 3x + 2 = 0$

As $\cos 3x = -2$ has no solutions, the only solutions are from

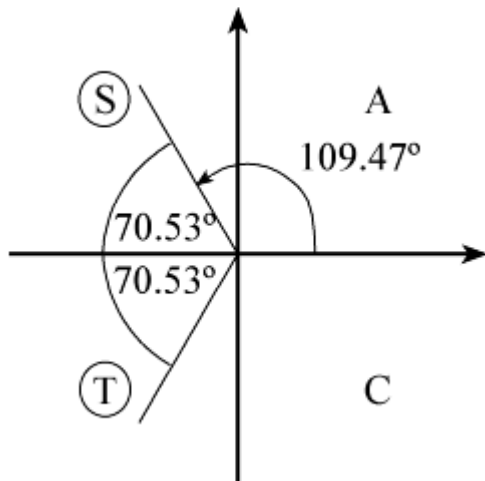
$$3 \cos 3x + 1 = 0 \text{ or } \cos 3x = -\frac{1}{3}$$

Let $X = 3x$

Solve $\cos X = -\frac{1}{3}$ in the interval $0 \leq X < 540^\circ$

The calculator solution is $X = 109.47^\circ$

As $\cos X$ is $-ve$, X is in the 2nd and 3rd quadrants.



Read off values of X in the interval $0 \leq X < 540^\circ$

$$X = 109.47^\circ, 250.53^\circ, 469.47^\circ$$

$$\text{So } x = \frac{1}{3}X = 36.49^\circ, 83.51^\circ, 156.49^\circ = 36^\circ, 84^\circ, 156^\circ \text{ (to the nearest integer)}$$

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Trigonometrical identities and simple equations

Exercise E, Question 17

Question:

Find, in degrees, the values of θ in the interval $0 \leq \theta < 360^\circ$ for which $2 \cos^2 \theta - \cos \theta - 1 = \sin^2 \theta$.
Give your answers to 1 decimal place, where appropriate.

[E]

Solution:

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$$2 \cos^2 \theta - \cos \theta - 1 = 1 - \cos^2 \theta$$

$$\Rightarrow 3 \cos^2 \theta - \cos \theta - 2 = 0$$

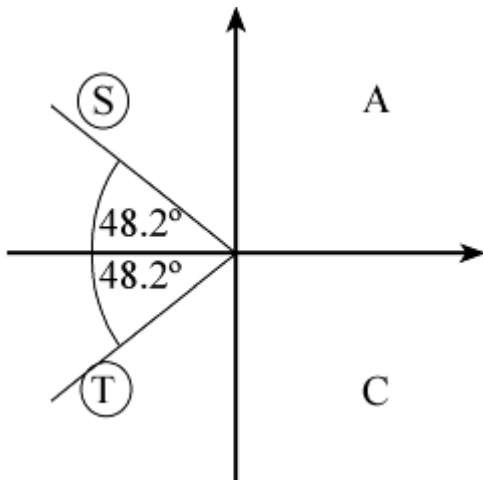
$$\Rightarrow (3 \cos \theta + 2)(\cos \theta - 1) = 0$$

So $3 \cos \theta + 2 = 0$ or $\cos \theta - 1 = 0$

For $3 \cos \theta + 2 = 0$, $\cos \theta = -\frac{2}{3}$

Calculator solution is 131.8°

As $\cos \theta$ is -ve, θ is in the 2nd and 3rd quadrants.



$$\theta = 131.8^\circ, 228.2^\circ$$

For $\cos \theta = 1$, $\theta = 0^\circ$ (see graph and note that 360° is not in given interval)

So solutions are $\theta = 0^\circ, 131.8^\circ, 228.2^\circ$

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Trigonometrical identities and simple equations

Exercise E, Question 18

Question:

Consider the function $f(x)$ defined by

$$f(x) \equiv 3 + 2 \sin(2x + k)^\circ, \quad 0 < x < 360$$

where k is a constant and $0 < k < 360$. The curve with equation $y = f(x)$ passes through the point with coordinates $(15, 3 + \sqrt{3})$.

(a) Show that $k = 30$ is a possible value for k and find the other possible value of k .

(b) Given that $k = 30$, solve the equation $f(x) = 1$.

[E]

Solution:

(a) $(15, 3 + \sqrt{3})$ lies on the curve $y = 3 + 2 \sin(2x + k)^\circ$

$$\text{So } 3 + \sqrt{3} = 3 + 2 \sin(30 + k)^\circ$$

$$2 \sin(30 + k)^\circ = \sqrt{3}$$

$$\sin\left(30 + k\right)^\circ = \frac{\sqrt{3}}{2}$$

A solution, from your calculator, is 60°

So $30 + k = 60$ is a possible result

$$\Rightarrow k = 30$$

As $\sin(30 + k)$ is +ve, answers lie in the 1st and 2nd quadrant.

The other angle is 120° , so $30 + k = 120$

$$\Rightarrow k = 90$$

(b) For $k = 30$, $f(x) = 1$ is

$$3 + 2 \sin(2x + 30)^\circ = 1$$

$$2 \sin(2x + 30)^\circ = -2$$

$$\sin(2x + 30)^\circ = -1$$

Let $X = 2x + 30$

Solve $\sin X^\circ = -1$ in the interval $30 < X < 750$

From the graph of $y = \sin X^\circ$

$$X = +270, 630$$

$$2x + 30 = 270, 630$$

$$2x = 240, 600$$

$$\text{So } x = 120, 300$$

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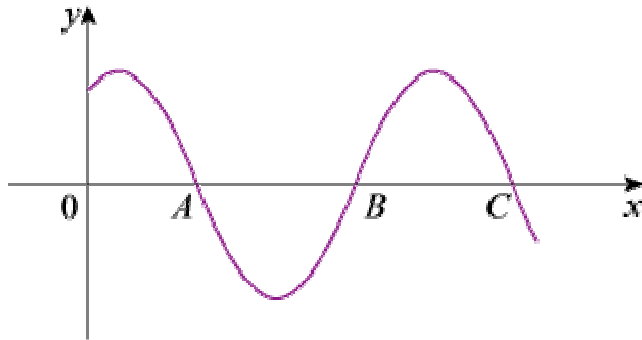
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Trigonometrical identities and simple equations

Exercise E, Question 19

Question:

- (a) Determine the solutions of the equation $\cos (2x - 30)^\circ = 0$ for which $0 \leq x \leq 360$.



- (b) The diagram shows part of the curve with equation $y = \cos (px - q)^\circ$, where p and q are positive constants and $q < 180$. The curve cuts the x -axis at points A , B and C , as shown.

Given that the coordinates of A and B are $(100, 0)$ and $(220, 0)$ respectively:

- Write down the coordinates of C .
- Find the value of p and the value of q .

[E]

Solution:

- (a) The graph of $y = \cos x^\circ$ crosses x -axis ($y = 0$) where $x = 90, 270, \dots$

Let $X = 2x - 30$

Solve $\cos X^\circ = 0$ in the interval $-30 \leq X \leq 690$

$X = 90, 270, 450, 630$

$2x - 30 = 90, 270, 450, 630$

$2x = 120, 300, 480, 660$

So $x = 60, 150, 240, 330$

- (b) (i) As $AB = BC$, C has coordinates $(340, 0)$

(ii) When $x = 100$, $\cos (100p - q)^\circ = 0$, so $100p - q = 90$ ①

When $x = 220$, $220p - q = 270$ ②

When $x = 340$, $340p - q = 450$ ③

Solving the simultaneous equations ② - ①: $120p = 180 \Rightarrow p = \frac{3}{2}$

Substitute in ①: $150 - q = 90 \Rightarrow q = 60$

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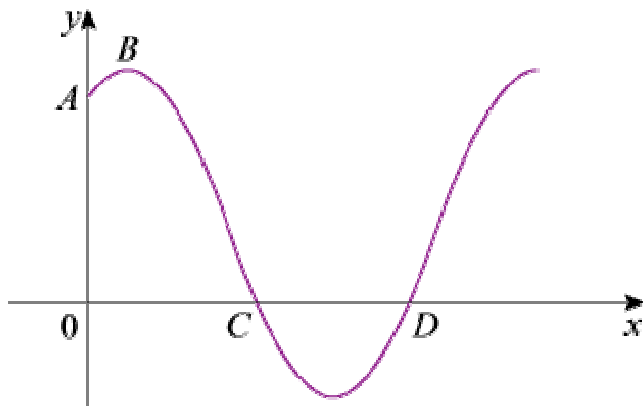
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Trigonometrical identities and simple equations

Exercise E, Question 20

Question:

The diagram shows part of the curve with equation $y = f(x)$, where $f(x) = 1 + 2 \sin(px^\circ + q^\circ)$, p and q being positive constants and $q \leq 90$. The curve cuts the y -axis at the point A and the x -axis at the points C and D . The point B is a maximum point on the curve.



Given that the coordinates of A and C are $(0, 2)$ and $(45, 0)$ respectively:

- Calculate the value of q .
- Show that $p = 4$.
- Find the coordinates of B and D .

[E]

Solution:

(a) Substitute $(0, 2)$ is $y = f(x)$:

$$2 = 1 + 2 \sin q^\circ$$

$$2 \sin q^\circ = +1$$

$$\sin q^\circ = +\frac{1}{2}$$

As $q \leq 90$, $q = 30$

(b) C is where $1 + 2 \sin(px^\circ + q^\circ) = 0$ for the first time.

$$\text{Solve } \sin \left(px^\circ + 30^\circ \right) = -\frac{1}{2} \text{ (use only first solution)}$$

$$45p^\circ + 30^\circ = 210^\circ \quad (x = 45 \text{ at } C)$$

$$45p = 180$$

$$p = 4$$

(c) At B $f(x)$ is a maximum.

$$1 + 2 \sin(4x^\circ + 30^\circ) \text{ is a maximum when } \sin(4x^\circ + 30^\circ) = 1$$

$$\text{So } y \text{ value at } B = 1 + 2 = 3$$

For x value, solve $4x^\circ + 30^\circ = 90^\circ$ (as B is first maximum)

$$\Rightarrow x = 15$$

Coordinates of B are $(15, 3)$.

D is the second x value for which $1 + 2 \sin (4x^\circ + 30^\circ) = 0$

$$\text{Solve } \sin (4x^\circ + 30^\circ) = -\frac{1}{2} \text{ (use second solution)}$$

$$4x^\circ + 30^\circ = 330^\circ$$

$$4x^\circ = 300^\circ$$

$$x = 75$$

Coordinates of D are $(75, 0)$.

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